Formal Verification of Energy Consumption for an EEG Monitoring Wireless Body Area Sensor Network

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***Abstract*—Wireless Body Area Sensor Networks (BASNs) are increasingly gaining notable attention in the domain of real-time and non-invasive human health care due to their cost-effectiveness. Minimizing their energy consumption under given data delay and distortion constraints is considered to be the most critical design factor for BASNs and specialized algorithms are developed for finding optimal parameters for minimizing the energy levels for BASNs. The optimization criteria are usually obtained based on the energy, delay and distortion relationships for the given BASN using paper-and-pencil proof and the performance of optimal parameter-finding algorithms is usually analyzed using simulation techniques. Due to the informal nature of paper-and-pencil proofs and simulation, 100% accuracy can never be ascertained, which is a severe limitation considering the safety-critical nature of BASNs. To overcome this limitation, we propose to use higher-order-logic theorem proving to conduct these analyses. As a first step towards this direction, this paper presents the higher-order-logic formalization of the commonly used mathematical relationships for energy consumption, data delay deadlines and distortion threshold constraints for an EEG monitoring BASN. These relationships can in turn be used for devising the optimization problem for the given BASN configuration.**

***Index Terms*—Wireless Body Area Sensor Network, Theorem Proving, Higher-order Logic**

I. Introduction

The global life expectancy, which was estimated at 65 years in 2000-2005, is expected to steadily rise to reach 75 years by 2045-2050 [24]. In this direction, Wireless Body Area Sensor Networks (BASNs), an amalgam of wireless and Body Area Networks [10], is an innovative methodology for real time and continuous monitoring of physiological activities, such as health status and motion patterns. Health-care BASNs are composed of tiny computing and communicating devices that allow us to monitor vital signs, such as temperature, heart-rate, electrocardiogram (ECG) and electroencephalogram (EEG) signals at real-time. These measurements are shared with a coordination node, where this information is automatically processed and automatic decisions are made or a health-care professional is called if required.

Due to the miniature size of the BASN nodes, they cannot carry large batteries and thus the minimization of their energy consumption is of utmost importance in this domain. Therefore, all the nodes are usually kept in the idle or sleep mode and a node becomes active upon receiving the information request from the central coordination node only. Reducing the communication time slots usually results in reducing the energy consumption but on the other hand may increase the communication delays. Similarly, compression ratio used for the BASN communication also affects the energy and the distortion thresholds [10]. Finding the most suitable values of flexible parameters, like transmitted data rate, compression ratio and wavelength filter length, to minimize energy consumption under some given delay and distortion constraints is a challenging optimization problem and various algorithms are used for solving this issue. Traditionally, these algorithms are developed by verifying the mathematical relationships for energy, delay and distortion of the given BASNs using paper-and-pencil proof methods. Once developed, the performance of these algorithms is judged using computer simulations. However, due to error-prone nature of paper-and-pencil proofs and the non-exhaustive characteristic of computer simulations for conducting analysis of continuous systems, the obtained parameters may not guarantee the predicted energy consumption, which is a severe limitation considering the safety-critical nature of human vital sign monitoring since a wrong estimate of energy consumption of the nodes may endanger human life.

Given the dire need of accuracy in the BASNs used for human health, formal methods have been explored for their verification. The notable contributions in this regard include the functional verification of BASNs protocols using model checking [23] and the recently conducted formal analysis of safety aspects of BASNs [1]. Most of the formal verification work related to sensor networks deals with the functional verification of generic wireless sensor network (WSN) algorithms. For example, RT-Maude is used to analyze the Optimal Geographical Density Control (OGDC) algorithm [20] by verifying its network coverage intensity and lifetime and the PVS theorem prover has been used to develop a generic functional verification framework for WSNs [6]. Similarly, model checking is used to verify WSNs security aspects using the SLEDE framework [13]. Formal methods have also been used for the performance analysis of WSNs. For example, the probabilistic model checker PRISM has been used to analyze the medium access control (MAC) protocols S-MAC [5] and ECo-MAC [25] for WSNs. Similarly, the HOL theorem prover has been used to analyze the performance of a scheduling algorithm for WSNs [11]. However, to the best of our knowledge, no formal analysis of energy consumption of BASNs has been done so far, which is the scope of the current paper.

We propose to formally verify the mathematical relationships for energy, delay and distortion of the given BASNs using higher-order-logic theorem proving. The high expressiveness of higher-order logic [9] allows us to capture the behavior of these relationships in their true form and interactive theorem proving [15] guarantees soundness of the verified results. The relationships between energy consumption, data delay and distortion are continuous in nature and thus cannot be accurately analyzed using the automata based formal methods, like model checking. Our formally verified results can then be used to determine the parameters that guarantee optimal energy consumption of BASN algorithms across the Application, MAC and Physical layers while considering the data delay deadlines and distortion threshold constraints.

As a first step towards the proposed direction of research, this paper presents the formal verification of the mathematical relationships of energy consumption, delay and distortion, given in [2] for a BASN that is used for EEG monitoring. The main motivation of choosing these relationships is that they allow dynamic power adaptation based on wireless link quality in the sensor and coordinator node communication for the given EEG monitoring application and thus most of the BASN configurations can be modeled as a subset of these advanced relationships.

The rest of the paper is organized as follows: A brief introduction to higher-order-logic theorem proving and the HOL theorem prover is given in Section II to facilitate the understanding of the paper for the BASN community. The energy, delay and distortion related formalization for the three network layers, i.e., Physical, MAC and Application, for the EEG monitoring BASN is then presented in Section III. Finally, Section IV concludes the paper.

II. Preliminaries

In this section, we give a brief introduction to theorem proving in general and the HOL theorem prover in order to facilitate the understanding of the rest of the paper.

A. Theorem Proving

Theorem proving is a widely-used formal method [17] and has been extensively used for rigorous verification of many safety-critical systems. In the theorem proving based verification, a mathematical model of the given system is expressed using a formal logic [19], such as propositional, second or Higher-order, and the properties of interest are then verified by deductive reasoning using a theorem prover, such as HOL [16], Coq [7] and PVS [21].

Theorem proving can be categorized into two main categories, i.e., *automated* and *interactive*. Automated theorem provers work with decidable logic, such as propositional logic, only and various decision procedures are thus utilized to search for the proof of given formula in the theory. However, there are very few systems that can be completely described using decidable logic. For example, continuous behaviors, involving *real* variables, cannot be described in propositional logic. For such cases, higher-order logic is used and formal reasoning about such models requires interactive theorem provers, where explicit user guidance is required to build the proof of a theorem in a computer. The ability to use a wide range of logic in the verification process makes theorem proving a highly expressive and flexible technique for formal verification.

The energy, delay and distortion relationships for BASN involve *real* numbers and *transcendental* functions, which cannot be modeled using propositional or first-order logic and thus we use higher-order logic in this paper.

B. HOL Theorem Prover

HOL, i.e., a descendant of the HOL88 family of theorem provers, is an interactive theorem prover, developed by Mike Gordon at Cambridge University [16]. HOL has been widely utilized for the formalization of mathematical theories and formal verification of hardware, software and communication protocols. HOL provides a platform for secure and sound formal verification and specification of system under consideration. Secure theorem proving is ensured due to its small core, which consists of 5 axioms and 8 inference rules, which are implemented as ML functions. A new theorem can only be proved by using the basic axioms, inference rules and the already proved theorems. Verified theorems can be stored for future use as a HOL theory, which is a computer file. These theories can be loaded by the HOL users in their HOL sessions and thus utilizes in the verification of new theorems. HOL supports both backward and forward proof methods. The former consists of using inference rules to deduce the proof of the desired theorem, whereas, the later consists of breaking the main goal into sub-goals using HOL tactics, which are special ML functions for generating sub-goals from a main goal.

For the brevity of content and concepts, used in this paper, Table I provides a list of HOL symbols and functions along with their mathematical equivalent, used in practice.

III. Energy, Delay and Distortion Relationships

In BASNs, each one of the sensor nodes communicates its corresponding constraints, i.e., the distortion thresholds and

TABLE I
HOL SYMBOLS AND FUNCTIONS

HOL Symbol	Standard Symbol	Meaning
$\wedge$	*and*	Logical *and*
$\vee$	*or*	Logical *or*
$\neg$	*not*	Logical *negation*
`λx.t`	$\lambda x.t$	Function that maps x to $t(x)$
`num`	$\{0, 1, 2, \ldots\}$	Positive Integers data type
`real`	All Real numbers	Real data type
`SUC n`	$n + 1$	Successor of a *num*
`rpow x y`	x^y	Power with real exponent

delay deadlines, and the channel conditions. Upon receiving this information, the coordination (or the master) node determines the most optimal transmission rate, modulation and energy for every communicating sensor node for a given bit error ratio (BER). It is important to note that the sensor nodes are usually power constrained but the coordination node may not have to meet very tight power budgets and that is the reason why the main emphasis of the following analysis is on optimizing the transmission energy of the sensor nodes.

The data rate r_i that can be transmitted using a single link i exhibits the following mathematical relationship with the received signal to noise ratio (SNR) γ, the channel bandwidth ω and $k = -1.5/log_{10}(5 * BER)$ [12] where BER represents the bit-error rate.

$$r_i = \omega.log_2(1 + k\gamma) \quad (1)$$

This relationship can be formalized in the HOL theorem prover as follows:

Definition 1: *Bit Error Rate Constant: k*

```
⊢ ∀ b. ber_k b =     -1.5
                 ─────────────────
                  log 10 (5 * b)
```

The HOL function `log` accepts a base x and the argument d for which the logarithm value is required and returns $log_x(d)$.

Definition 2: *Data Transmission Rate Relationship*

```
⊢ ∀ r w b g. data_tr_rate_rel r w b g =
   (r = w * log 2 (1 + (ber_k b) * g)
```

The function `data_tr_rate_rel` accepts four variables: r represents the data transmission rate, w represents the channel bandwidth, b represents the bit error rate and g represents the SNR. From these two definitions, we can formally verify the following expression for the SNR (γ):

Theorem 1: *Signal to Noise Ratio (Upper bound on b)*

```
⊢ ∀ g w b. (0 < g) ∧ (0 < w) ∧ (0 < r) ∧
   (data_tr_rate_rel r w b g) ∧ (0 < b) ∧
   (b < min (1 / 5) (10 rpow (1.5 * g) / 5)) ⇒
  g = 2 rpow (r / w) - 1
      ──────────────────
           ber_k b
```

where the HOL function `rpow` models the power function with real exponents. We primarily verified the above theorem by first verifying the logarithm property: `∀ b d. 0 < d ∧ 1 < b ⇒ (b rpow (log b d) = d)` and the arithmetic relationship: `∀ a b c. c < min a b ⇒ (c < a ∧ c < b)`. The assumptions of Theorem 1 are based on the physical aspects and represent the constraints under which the conclusions of these theorems hold. The first three assumptions ensure that the SNR `g`, the channel bandwidth `w` and the data transmission rate `r` are all greater than zero. The next assumption defines the relationship of the data transmission rate `r` with the rest of the argument parameters, as given in Definition 2. The last two assumptions provide the interval of the bit error rate `b` under which the relationship holds, i.e., $(0, min(\frac{1}{5}, \frac{10^{(1.5\gamma)}}{5}))$. It is important to note that the conclusion of the above theorem, i.e., the SNR relationship, was reported in the paper-and-pencil based analysis of the same problem [2], however, the allowable values of the bit error rate b for the validity of this relationship were not mentioned. We initially tried to verify this SNR relationship without this constraint but the theorem prover actually guided us to deduce these constraints, since the expression does not hold for all values of the variable `b`. This clearly indicates the usefulness of using formal methods in analyzing a safety-critical application like BASNs since a design without catering for such missing constraints may lead to catastrophic consequences.

Another widely used parameter of BASNs is the received power P_r, which is described in terms of the SNR, the channel noise spectral density N_0 and the channel bandwidth ω:

$$P_r = \gamma.N0.\omega \quad (2)$$

Now, the transmitted power P_t can be expressed as

$$P_t = \frac{P_r}{\alpha};\ \alpha = \frac{g_t.g_r.\lambda^2}{(4.\pi.d)^2} \quad (3)$$

where g_t is the transmit antenna gain, g_r is the receive antenna gain, λ is the signals wavelength, d is the distance between the sensor and coordination nodes and α is the overall path loss [22]. Based on the above definitions the transmitted energy while using the EEG compression techniques can be expressed as follows [2]:

$$E_t = \frac{P_t.l_i.(1 - C_{R_i})}{r_i} \quad (4)$$

where l_i is data length for node i and C_{R_i} is the compression ratio for node i. It is important to note here that the amount of data transferred is inversely proportional to the EEG compression ratio and therefore as C_R increases the corresponding transmitted energy also decreases [2].

The HOL formalization of Equations(2 - 4) is as follows:

Definition 3: *Received Power*

```
⊢ ∀ g N0 w. rec_pow g N0 w = g * N0 * w
```

Definition 4: *Path Loss*

```
⊢ ∀ gt gr lambda d.
  path_loss gt gr lambda d =
                     (gt * gr * (lambda pow 2))
                     ──────────────────────────
                        (4 * pi * d) pow 2
```

Definition 5: *Transmitted Power*

```
⊢ ∀ g N0 w gt gr lambda d.
  trans_pow g N0 w gt gr lambda d =
                        (rec_pow g N0 w)
                  ──────────────────────────
                  (path_loss gt gr lambda d)
```

Definition 6: *Transmitted Energy*

```
⊢ ∀ g N0 w gt gr lambda d l cr r.
  trans_en g N0 w gt gr lambda d l cr r =
  (trans_pow g N0 w gt gr lambda d) * l * (1 - cr)
  ------------------------------------------------
                        r
```

The HOL function power pow used in Definition 4 models the real power function with a positive integer exponent. Another important parameter related to the physical layer communication in BASN is the channel gain x_i, which is mathematically expressed as [4]:

$$x_i = \frac{k.\alpha}{N_0.\omega}|h_i|^2 \tag{5}$$

where $|h_i|$ represents the fading channel magnitude for the link i. This definition can be formalized in HOL as follows:

Definition 7: Channel gain

```
⊢ ∀ b N0 w gt gr lambda d h.
  ch_gain b N0 w gt gr lambda d h =
  (ber_k b) * (path_loss gt gr lambda d)
  -------------------------------------- *
                 N0 * w
  (abs (h) pow 2)
```

Besides the transmitted energy, the second most important factor that contributes towards the overall power dissipation in a BASN is the encoding energy, which can be obtained using the following expression [18]:

$$E_p = c_1.(1 - C_R) + c_2.F \tag{6}$$

where c_1 and c_2 are constants that depend on the used system model, C_R is the compression ration and F is wavelet filter length. This relationship can be formalized in HOL as follows:

Definition 8: *Encoding Energy*

```
⊢ ∀ c1 c2 F cr.
  encod_en c1 c2 F cr = c1 * (1 - cr) + c2 * F
```

The source coding distortion D_p exhibits the following relationship with the compression technique:

$$D_p = c_3.e^{-c_4.C_R} + c_5.e^{c_6.C_R} - c_7.e^{c_8.F} - 2 \tag{7}$$

where c_3 to c_8 are constants that depend on the used system model. This equation can be formalized in HOL as follows:

Definition 9: *Source Coding Distortion*

```
⊢ ∀ c3 c4 c5 c6 c7 c8 F cr.
  sc_dist c3 c4 c5 c6 c7 c8 F cr =
  c3 * exp (-c4 * cr) + c5 * exp (c6 * cr) -
  c7 * exp (c8 * F) - 2
```

Where the exp function models the exponential function in HOL [14].

For an overall energy consumption reduction, a variable-length Time Division Multiple Access (TDMA) scheme is used for BASNs [26] where each communicating sensor node gets a time slot of variable duration depending on the application requirements. The time period for this slot in calculated using the data length l_i, data transmission rate r_i and the compression ratio C_{R_i} [2]:

$$t_i = \frac{l_i.(1 - C_{R_i})}{r_i} \tag{8}$$

This can be formalized in HOL as follows:

Definition 10: *Time Period*

```
⊢ ∀ l r cr.
  tm_period l r cr = l * (1 - cr)
                     ------------
                          r
```

Now, using the above definitions and Theorem 1, we formally verified the following simplified relationship for the transmitted energy:

Theorem 2: *Alternate Expression for Transmitted Energy*

```
⊢ ∀ b g N0 w gt gr lambda d l cr r h.
  (0 < g) ∧ (0 < w) ∧ (0 < r) ∧
  (data_tr_rate_rel r w b g) ∧
  (0 < b) ∧
  (b < min (1 / 5) (10 rpow (1.5 * g) / 5)) ∧
  (0 < gt) ∧ (0 < gr) ∧
  (0 < lambda) ∧ (0 < d) ∧
  (0 < N0) ∧ ¬(h = 0) ⇒
   trans_en g N0 w gt gr lambda d l cr r
   = (1 - cr) * l * (2 rpow (r / w) - 1)
     -----------------------------------
     r * ch_gain b N0 w gt gr lambda d h
```

The first six assumptions are the same as Theorem 1. The remaining six represent the physical constraints on the variables: The transmit gt and receive gr antenna gains, the signal wavelength lambda, the distance between the sensor and coordination nodes d and the channel noise spectral density N0 have to be greater than 0. While the fading channel magnitude h cannot be 0. The proof of Theorem 2 is primarily based on arithmetic reasoning.

The definitions and theorems, presented in this section, formally describe the energy, delay and distortion relationships for a BASN used for EEG. This formalization can now be used to formally describe the optimization problem for the given BASN. The main difference between this formally verified result and the corresponding one found using paper-and-pencil proof methods [2] is the explicit presence of all the assumptions in Theorems that are required for their validity. It is important to note that, some of these conditions are not considered in the optimization criteria, given in the paper-and-pencil proof based verified optimization criteria for the given BASN [2]. The identification of these corner cases is the main strength of the proposed approach since failing to meet any one of these criteria may lead to the design flaws that may in turn result in life threatening situations. As a more realistic case-study for our formalization, we plan to formally analyze the optimality of the design parameters for health care BASNs, reported in [3].

IV. Conclusion

BASNs are increasingly being advocated to be used in e-health applications. Due to the safety-critical nature of the application, correct analysis and design for BASN based e-health applications is a dire need. Most of the existing work related to the formal verification of BASNs is concentrated towards the functional analysis of their algorithms and some security related issues. In this paper, we propose to use higher-order-logic theorem proving for verifying the energy relationships for BASNs.

The design and analysis of optimal parameter finding algorithms for BASNs is usually done in a two-step process.

Firstly, the behavior of the system is mathematically modeled and an optimization problem is formulated using paper-and-pencil proof methods. Secondly, the algorithms are developed to solve these problems and the efficiency of these algorithms is judged based on computer simulations. This paper primarily targets the first step and provides the formalization of energy, delay and distortion relationships for a BASN that is used for an EEG monitoring application, which is selected due to its widespread nature. Our results were found to be exhaustive in terms of the identification of all constraints for the validity of the analysis. These formally verified relationships can in turn be used for determining the optimal parameters to minimize the energy consumption of the given BASN. This step is under investigation and the main challenge is the higher-order-logic formalization of optimization problem solving methods, such as the interior-point methods for solving convex problems [8].

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