Formal Periodic Steady-State Analysis of Power Converters in Time-Domain

Asad Ahmed
School of Electrical Engineering and Computer Science (SEECS)
National University of Sciences and Technology (NUST), Islamabad, Pakistan
asad.ahmed@seecs.nust.edu.pk

Osman Hasan
School of Electrical Engineering and Computer Science (SEECS)
National University of Sciences and Technology (NUST), Islamabad, Pakistan
osman.hasan@seecs.nust.edu.pk

Ammar Hasan
School of Electrical Engineering and Computer Science (SEECS)
National University of Sciences and Technology (NUST), Islamabad, Pakistan
ammar.hasan@seecs.nust.edu.pk

Abstract

Time-domain based periodic steady-state analysis is an indispensable component to analyze switching functionality and design specifications of power electronics converters. Traditionally, paper-and-pencil proof methods and computer-based tools are used to conduct the time-domain based steady-state analysis of these converters. However, these techniques do not provide an accurate analysis due to their inability to model and analyze continuous behaviors exhibited by the power electronics converters. On the other hand, an accurate analysis is direly needed in many safety and cost-critical power electronics applications, such as biomedical, hybrid electric vehicles, and aerospace engineering. To alleviate the issues pertaining to the above-mentioned techniques, we propose a methodology, based on higher-order-logic theorem proving, to conduct the time-domain based steady-state analysis of power electronics converters in this paper. The proposed methodology is primarily based on a formalized switching function analysis technique, ordinary linear differential equations and steady-state conditions of the systems. To illustrate the usefulness of proposed formalization, we present the formal time-domain steady-state analysis of a commonly used DC-DC Buck converter.

1 Introduction

Power electronics converters are an integral part of, almost, every realizable electrical/electronics system, as a power processing stage, to meet their power requirements [10]. These systems are typically composed of semiconductor devices, like switches, energy storage and dissipative elements, i.e., inductors, capacitors, and resistors, and integrated circuits for control. Generally, periodic steady-state analysis is a mandatory preprocessing step for the small-signal analysis, which is used to evaluate the performance of the converter. Moreover, time-domain based analysis is necessary for the study of the switching functionality, which is central to the power conversion operation of the converters [10]. However, switching is a highly non-linear phenomenon and therefore leads to significant modeling, analysis and design challenges of these systems.

Traditionally, paper-and-pencil proof methods or computer-based numerical techniques are used to perform the time-domain based steady-state analysis of the power electronics systems. The paper-and-pencil proofs are usually based on many assumptions, such as small-ripple approximations, and averaging techniques to linearize the nonlinear behavior of the systems to analyze the systems in steady-state [10]. These linearized models, expressed as ordinary linear differential equations, are then simulated using a variety of computer based simulation tools, such as MATLAB Simulink, Saber, PSpice, to evaluate the performance of the power electronics systems. Generally, these computer based simulation tools use discretized time or frequency domain models of the systems and numerical integration methods [7] for solving the differential equations of the converters [8]. Therefore, the above-mentioned techniques cannot ascertain an accurate and reliable analysis of the power converters due to inherent approximation based nature of these techniques. For example, the accuracy of paper-and-pencil proof methods is usually limited by the underlying approximate linearized model. On the other hand, the nonlinear analysis is, mathematically, not tractable and due to human involvement is highly likely error prone. Similarly, the numerical methods employed in the simulation techniques, based upon the discretization of time or frequency, lead to truncation errors and also cannot accurately model the hybrid behavior, i.e., continuous behavior driven by discrete events, exhibited by power converters [22]. To address this issue, computer algebra systems, which are software programs for the symbolic processing of mathematical expressions, are also employed for the analysis of such systems [16]. However, the symbolic processing is based on the unverified program codes, and therefore prone to bugs [21]. Thus, given the aforementioned inaccuracies, these traditional techniques should not be relied upon for the analysis of power electronics systems, especially when they are used in safety-critical areas, such as implantable medical devices [3] and automotive industry [9], and mission-critical areas, such as aerospace engineering [13], where bugs may lead to heavy monetary or human life loss.

In recent years, formal methods have been extensively employed for the accurate analysis of a variety of hardware and software systems. The transfer function of DC-DC converters has been verified [6] in the frequency domain using higher-order-logic theorem proving based on the signal flow graph and Mason's gain formula. The transfer function is then used to reason about the efficiency, stability and resonance of pulse width modulation push-pull DC-DC converter and 1-boost cell DC-DC converter. However, the nature of formalization does not permit to reason about the interesting features of switch, which is a key element of power electronic converters. Model checking has also been used for the analysis of the DC-DC Buck circuit [18] [20] using a hybrid automaton equivalent model of circuit to verify the reachability and safety properties of the circuit. However, the state-based modeling of the circuit does not allow to describe the exact continuous behavior of power converters circuits. Moreover, the state-space explosion issues also limit the scope of model checking for the verification of continuous and hybrid systems. To the best of our knowledge, there is no formal approach in the literature that explicitly allows us to verify the nonlinear aspects pertaining to the modeling and time-domain based steady-state analysis of power electronics systems.

The main motivation of this paper is to develop a formal logical framework for the time-domain based steady-state analysis of power converters. The main challenge in this direction is to be able to model and analyze the continuous structural or topological changes under the switching action [5], which are usually modeled using the Heaviside step function [1], i.e.,

$$u(t) = \begin{cases} 1 & 0 < t \\ 1/2 & t = 0 \\ 0 & t < 0 \end{cases} \tag{1}$$

The topological changes deter the explicit use of conventional circuit analysis techniques, such as mesh and node analysis, for investigating the implementation of the circuit by using the behavior of its individual components and its overall behavior [17]. Another notable consequence is that the switching action introduces piecewise functions, which are also expressed in terms of the Heaviside step function, in the analysis that in turn cannot be analyzed using linear mathematical techniques based on the Riemann integral theory, such as differential chain rule and integration by part. To tackle the former issue, we propose to use the switching function technique [17], which is a commonly used circuit analysis technique that allows to incorporate the topological changes of the circuit in the analysis. We tackled the piecewise nature of the functions in our formal framework by using the Gauge or Henstock-Kurzweil integral [15]. The Gauge integral is characterized by the Gauge function for the tagged division of an interval over which the function is to be integrated. This simple, but novel, alteration allows us to integrate the functions with countable singularities or the functions that are continuous but not differentiable everywhere on the given interval. It, particularly, supported us in the formal verification of an interesting notion of the Heaviside step function as a generalized function [14] which is widely used to describe discontinuous phenomena in physics and engineering disciplines. As a generalized function, the Heaviside step function acts as an operator on a test function $f(x)$, which needs to be smooth everywhere, as:

$$\int_a^b h(x-c)f(x) = \int_c^b f(x) \qquad \forall\, a\, b\, c.\ a < c < b \tag{2}$$

The smoothness of test function also plays a pivotal role in the differentiation of the piecewise functions involving the Heaviside step function in the formal time-domain based periodic steady-state analysis of power converters.

Besides these foundations, the proposed formalization is based on the formalizations of linear ordinary differential equations and steady-state conditions. The homogeneous linear differential equations using real analysis have been formalized in HOL to model the cyber-physical systems [19]. In this paper, we have extended the logical framework, presented in [19], to the non-homogeneous linear differential equations using complex analysis to formally model the dynamic behavior of the power converters. We have used the multi-variable integral, differential, transcendental and topological theories to define the steady-state conditions due to the piecewise nature of the functions involved in the analysis.

The formalization in this paper is done using the HOL-Light theorem prover [11], which supports formal reasoning about higher-order logic. The main motivation behind this choice is the availability of reasoning support about multi-variable integral, differential, transcendental and topological theories [12], which are the foremost foundations required for the formalization of time-domain based steady-state analysis of power electronics systems.

The rest of the paper is organized as follows: We describe some preliminaries regarding the periodic steady-state analysis of power electronics converters in Section 2. In Section 3, we present the proposed methodology. The formalization of the switching function technique, ordinary differential equations and steady-state conditions in Section 4. We utilize this formalization to formally verify a Power converter circuit, i.e., DC-DC buck converter in Section 5. Finally, Section 6 concludes the paper.

2 Periodic Steady-state Analysis of Power Converters

Power converter circuits use continuous switching among different circuit configurations to achieve the desired power conversion, such as dc-dc, dc-ac, ac-dc and ac-ac. In each circuit configuration, also called mode or state of the converter, the behavior of the circuit variables can be expressed as differential equations with initial conditions from the previous mode at the switching instance. Therefore, the standard approach for the time-domain analysis of these converters consists of developing the differential equations for each mode of the circuit based on the Kirchoff's voltage or current laws to describe the dynamic behavior of these circuits.

Mathematically, the behavior of these systems can be described as:

$$\begin{aligned} H(t, y_1, y_1^1, ..., y_n^{m_n}) &= p(t) && t \in [t_{n-1}, t_n], n, m_n \in \mathbb{N} \\ y_n^k(t_n) &= y_{n-1}^k(t_{n-1}) && k \in \mathbb{N} \\ y_0^1(t_0) &= 0 \end{aligned} \tag{3}$$

Where, H and p are functions of an independent variable t, a dependent variable y_n and its m_n-th order derivative in the corresponding n-th mode, respectively. In power converters, the time is considered as an independent variable, whereas, the voltage or current of the energy storage components is considered as a dependent variable. The order, i.e., m_n, of an ordinary differential equation of the power converter, in the n-th mode, is determined by the number of energy storage elements constituting the mode. The function $p(t)$ is referred to as a non-homogeneous term, which can be zero or non-zero in the n-th mode, depending upon the presence of source in the n-th mode of a power converter. Initially, the value of dependent variable is considered zero, i.e., $y_0^1(t_0) = 0$, however, later on the value of the dependent variable in one mode becomes an initial value for the next mode, i.e., $y_n^k(t_n) = y_{n-1}^k(t_{n-1})$, when switching instance occurs. Whereas, k is the order of the derivative of the dependent variable evaluated at a specific time instance.

For the brevity of the notion, transient and steady-state time-domain behavior of a DC-DC power converter is presented in Fig. 1, base on the above-mentioned standard approach. DC-DC power converter circuits are designed to step-up or step down the dc voltage levels applied at their input. Fig. 1 shows the output behavior, y_t, of a DC-DC power converter under the switching action represented by a rectangular switch wave form, S_w.

In periodic steady-state, the dependent variables of a power converter circuit attain an equilibrium and repeat the behavior over a time period, T_p, constituting

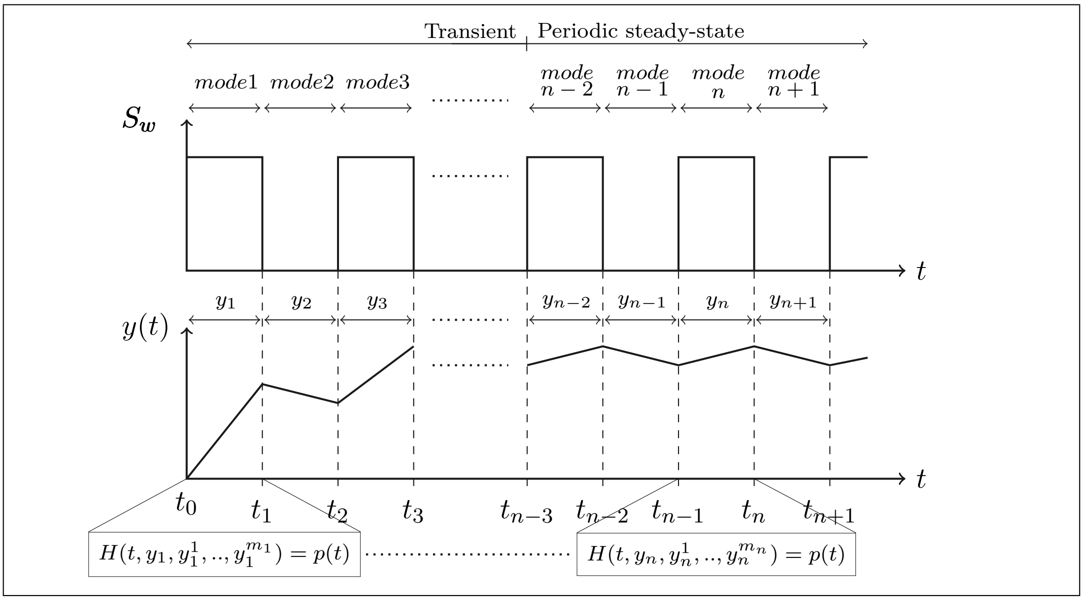

Figure 1: **Dynamic behavior of the output, $y(t)$, of a DC-DC power converter under switching action, represented by the switching wave form, S_w.**

l modes. Mathematically, the periodic steady-state behavior of a power converter over one time period, when $t \to \infty$, can be represented as:

$$\begin{aligned} H(t, y_n, y_n^1, ..., y_n^{m_n}) = p(t) \quad t \in T, T \in \bigcup_{i=1}^{l} \left[t'_{i-1}, t'_i\right], m_n, n, l \in \mathbb{N} \\ y^k(t'_0) = y^k(t'_0 + T_p) \quad T_p = t'_{max(i)} - t'_0, k \in \mathbb{N} \end{aligned} \tag{4}$$

Equation (4) reduces the problem to the identification of the modes in one time period, $T_p = t'_{max(i)} - t'_0$, of the circuit, which is the length of time over which the modes of a power circuit converter repeat themselves. The function y is a piecewise function defined over l modes. Whereas, $y^k(t'_0) = y^k(t'_0 + T_p)$ refers to the steady-state conditions of the system variable at reference switching time instances, t'_0, and T_p, and k represents the k-th order derivative of the variable.

Fig. 2 illustrates the behavior of the output of a DC-DC power converter in steady-state, which is mathematically modeled in Equation 4. The output, $y(t)$, of the converter exhibits a repetitive behavior over the time period T_p in l modes. In literature, waveforms of the dependent variable, y, are used for the periodic steady-sate analysis of the power converters by applying the principle of inductor

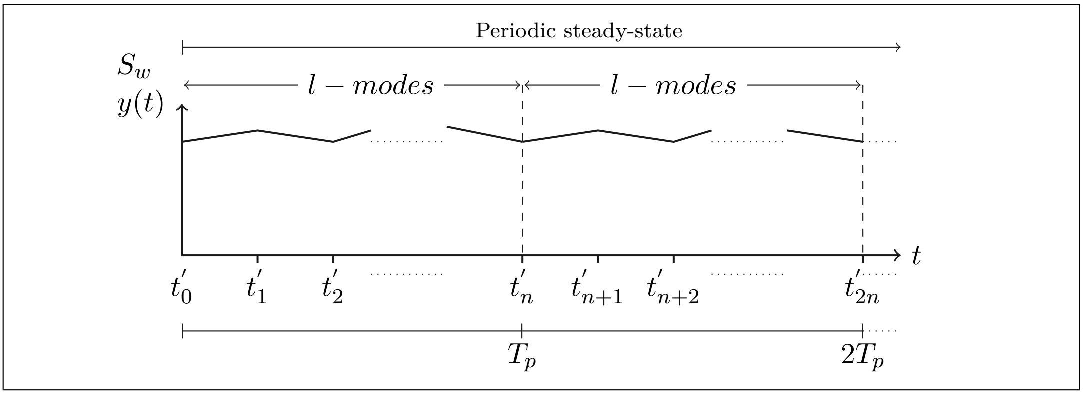

Figure 2: **Behavior of the output, $y(t)$, of a DC-DC power converter in Periodic steady-state.**

volt-second or capacitor-charge, along, with small-ripple approximations to reduce the complexity of the analysis by compromising the accuracy [10].

In this paper, we propose a logical framework for the formal verification of the periodic steady-state analysis of power converters in time domain, which are mathematically represented by Equation 4. The challenges to develop a logical framework for the formal verification of the aforementioned problem are two fold. Firstly, we intend to develop a higher-order logic formalization capable of incorporating the topological structural changes over the time period, i.e., $T \in \bigcup_{i=1}^{l} \left[t_{i-1}^{'}, t_i^{'}\right]$, thus, enabling us to formally model and reason about the implementation behavior of these circuits within the sound core of the HOL-Light theorem prover. Second we want to develop a formal library of foundations, including; differential equations, concepts from operational calculus described by Equation 2, to formally reason and verify the highly nonlinear behavior of the circuit variables involved in the formal periodic steady-state analysis of these circuits, in higher-order logic. The respective subsections of Section 4 address these challenges by presenting the formalization of switching function technique, differential equations and solution of these differential equations, respectively, to conduct the formal periodic steady-state analysis of power converters in the time-domain.

In the next section, we present the proposed methodology for the formal periodic steady-state analysis of the power converters, in a higher-order-logic theorem prover, i.e., HOL-Light.

3 Proposed Methodology

We propose to use higher-order-logic theorem proving, as shown in Fig. 3, in order to formally verify the power converters operating in the periodic steady-state. The first step in the proposed methodology is to build a formal model for the switching function technique and linear order differential equations to formally express the implementation and specification of power converter circuits, in higher-order logic. The proposed formal modeling of switching function technique is based on the formal definitions of an ideal semiconductor switch, energy storage and dissipative elements, and Kirchoff's current and voltage laws. Whereas, the formal modeling of the linear ordinary differential equation is used for the formal specification of the behavior of each mode of the power converter circuit. The aforementioned two formal models can then be used to formally assert and analyze the implementation of the circuits, as a theorem, using the sound core of HOL-Light. Moreover, the formal specification of ordinary linear differential equations is also used to formally

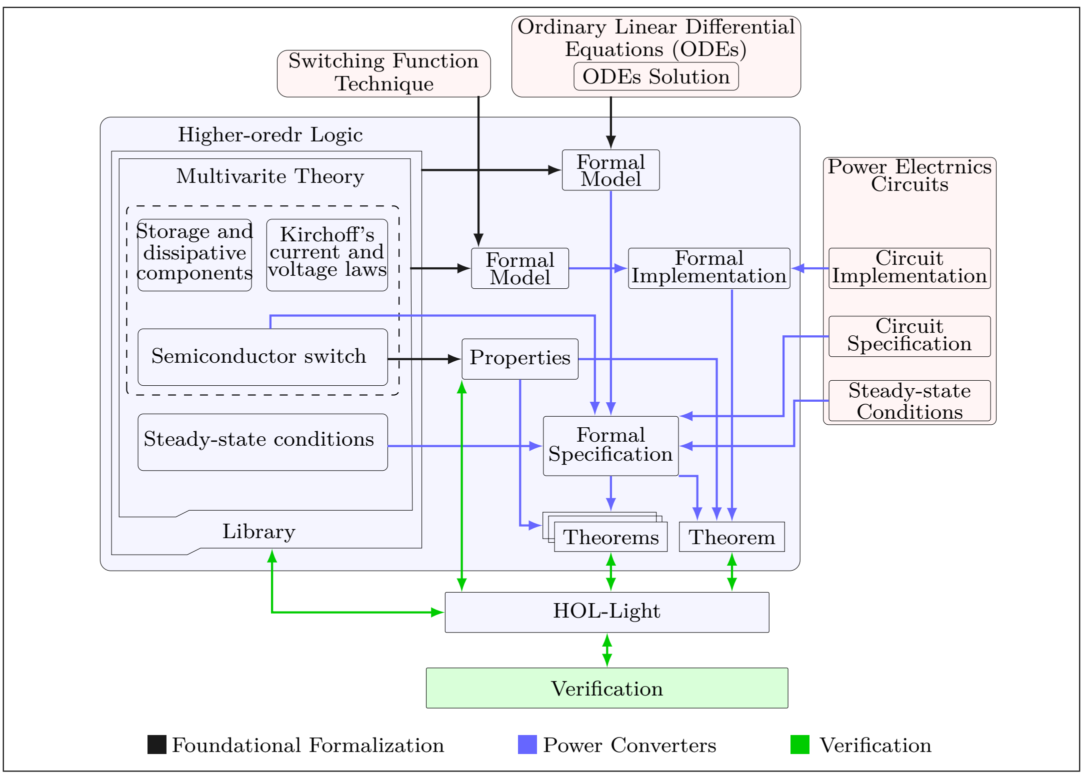

Figure 3: **Proposed Methodology**

verify the correctness of the solutions of these equations. As the steady-state analysis is based upon the formal modeling of the linear ordinary differential equations and their solutions, therefore, in the next step, we propose to formally define the steady-state conditions to conduct the formal analysis of power converters, as shown in Fig 3. These formal definitions, along with multi-variable theories of HOL-Light, are used to formally verify the theorems that are required to conduct the formal steady-state analysis of power converters. Finally, the switch is formalized using the Heaviside step function, and its related properties, such as integration and derivation of piecewise functions involving Heaviside step function, are formally verified. As the switching functionality plays the most vital role in characterizing the nonlinear behavior of the power converters therefore these formally verified properties are used in, almost, every aspect of the formalization and verification.

4 Foundational Formalizations

4.1 Formal Model of the Switching Function Technique

In power converter circuits, semiconductor devices such as, diodes, BJTs (bipolar junction transistors), MOSFETs (metal oxide semiconductor field effect transistors), IGBTs (insulated gate bipolar transistors) etc, are used for performing the switching operation. These semiconductor devices play a vital role in the development of reliable, cost-effective and highly efficient converters [4]. Although, these devices differ in their physics and physical properties, however, as a switch, their function is to connect or disconnect a path or subcircuit, in a converter circuit, to achieve the desired conversion. Therefore, the functionality of an ideal semiconductor device as a switch can be modeled using the Heaviside function, i.e., Equation (1), in HOL-Light:

Definition 1:
```
⊢ ∀ t.  semi_switch t = if t < &0 then &0 else
                             (if t = &0 then &1 / &2 else &1)
```

Definition 1 models the functionality of a semiconductor switch as a real value `1`, for connected status, and `0`, for disconnected status, in higher-order logic. Whereas, at the switching instance `t`, it has value `1/2`. The `&` is a typecasting operator in HOL-Light that maps a number to a real number. In our formalization, we use switch status or switching function to refer connected or disconnected switch.

The switching operation is central to the power converters functionality, however, it hinders the straightforward usage of the conventional circuit theory techniques, such as Kirchoff's voltage and current laws. The switching function technique relies

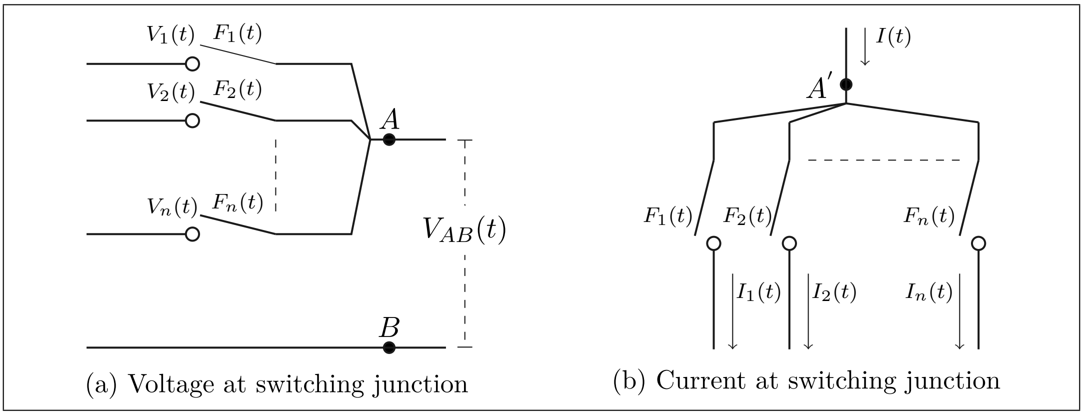

Figure 4: **Switching function technique**

on the superposition theorem of the voltage or current to express the behavior of these quantities in the presence of a switch in the circuit. It is based on the conceptualization of the switch as a modulating function for the input and output power. Based on this notion, the voltages and the currents in the presence of a switch component can be expressed as [1];

$$V_{AB}(t) = \sum_{i=1}^{n} V_i(t)F_i(t) \qquad n \in \mathbb{N} \tag{5a}$$

$$I_i(t) = I(t)\sum_{i=1}^{n} F_i(t) \qquad n \in \mathbb{N} \tag{5b}$$

Equation 5(a), describes voltage at the switch junction, in a mesh, in terms of switching functions. Fig. 4(a) is a pictorial representation of the concept, where n voltage sources are connected to a point, A, through n switches. The voltage, V_{AB}, is then the superposition of the input voltages, however, the contribution of each voltage is dependent upon the associated switching function. Similarly, Equation 5(b), describes the current at a node, $A^{'}$, which has n switches. Fig. 4(b) describes the situation where current, $I(t)$, is supplied to n paths of the circuit through n switches. Each path receives the fraction of total current depending upon its switch status, $F_n(t)$.

Voltages and currents at the switching junction in higher-order logic are defined, as:

Definition 2: $\vdash \forall$ `mod_lst volt_lst t.`

```
switch_volt mod_lst volt_lst t =
vsum (0..LENGTH mod_lst - 1) (λ n.  EL n volt_lst t * Cx (EL n mod_lst))
```

The function `switch_volt` describes the voltage at the switch junction using Equation 5(a). It accepts a list, `volt_lst`, which contains all the possible voltage drops at the switching junction, a list of modes, `mod_lst`, which contains the switch status or switching function for each mode, and `t` is the time, which indicates that this function is time dependent. Whereas, `Cx` is a HOL-Light function, which is used to map a real number, representing the switching function, to a complex number.

Definition 3: `⊢ ∀ mod_lst curr t.  switch_current mod_lst curr t =`
`curr t * vsum (0..LENGTH mod_lst - 1) (λ n.  Cx (EL n mod_lst))`

Definition 3 formally models the current at the switching junction using Equation 5(b). It accepts an argument `curr`, which represents the total supplied current to the switch junction, a list of modes, `mod_lst`, which contains the switch status or switching function for each mode, and `t`, which represents time.

To accomplish the formal modeling of the switching function technique, we also formalize the Kirchoff's voltage and current laws:

Definition 4: `⊢ ∀ vol_lst t.  kvl vol_lst t =`
`vsum (0..LENGTH vol_lst - 1) (λn.  EL n vol_lst t) = Cx (&0)`

Definition 5: `⊢ ∀ cur_lst t.  kcl cur_lst t =`
`vsum (0..LENGTH cur_lst - 1) (λn.  EL n cur_lst t) = Cx (&0)`

The `kvl` and `kcl` functions accept lists of type $(\mathbb{R} \rightarrow \mathbb{C})$, to express the behavior of the time dependent voltages and currents in the given power converter circuit and a time variable `t`. They return the predicates that guarantee that the sum of the voltages in a loop or sum of the currents at a node are zero for all the time instants.

The voltages and currents in Definitions 2 and 3 are piecewise functions due to switching action. We formally verified the result of Equation (2) to conduct the formal analysis involving such functions:

Theorem 1: `⊢ ∀ f a b c x.`
`A1:(∀t.  (λx.  f (x)) differentiable_on s) ∧`
`A2:∼(real_interval [a,b] = {}) ∧`
`A3:c ∈ [a, b]`
$\Rightarrow \int_a^b$ `(λx.  semi_switch x c ) * f (x)) =` $\int_c^b$ `(λx.  f (x))`

The Assumption `A1` ensures the differentiability of a test function, `f`, over `s`. Whereas, `s`:$(\mathbb{R} \rightarrow \mathbb{B})$ is a set-theoretic definition of the intervals in higher-order logic, over real numbers. For a given real interval [a,b], it represents all possible real intervals, which are subsets of the given real interval. Therefore, Assumption A1 ensures the differentiability of a test function over all subsets of the given real interval [a,b]. Assumptions A2 and A3 ensure that the interval is non-empty and point c lies within the interval [a, b]. The conclusion of the Theorem 1 formally verifies the affect of applying the Heaviside step function on a test function, i.e., changes the limit of integral. Theorem 1 is formally verified using the formal definition of Gauge integral and its properties, available in HOL-Light theorem prover. This formally verified result plays a very key role in the formal reasoning of the systems which exhibit nonlinear behavior, such as power converters circuits.

The above formalization enables us to formally model and analyze the nonlinear behavior exhibited by the power converters, due to switching action, in higher-order logic.

4.2 Ordinary Linear Differential Equation

An n^{th}-order ordinary linear differential equation can be represented as:

$$a_n(t)\frac{d^n y(t)}{dx} + a_{n-1}(t)\frac{d^{n-1}y(t)}{dx} + ... + a_0(t)y(t) = p(t) \tag{6}$$

We formalized the n^{th}-order derivative function in higher-order logic as follows:

Definition 6: ⊢ ∀ n f t. (n_vec_deri 0 f t = f t) ∧
(∀ n. n_vec_deri (SUC n) f t =
n_vec_deri n (λ t. vector_derivative f at t) t)

The function n_vec_deri accepts a positive integer n that represents the order of the derivative, the function f:($\mathbb{R} \rightarrow \mathbb{C}$) that represents the complex-valued function that needs to be differentiated, and the variable t:($\mathbb{R}$) that is the variable with respect to which we want to differentiate the function f. It returns the n^{th}-order derivative of f with respect to t. Now, based on this definition, we can formalize the left-hand side (LHS) and right-hand side (RHS) of Equation (6) in HOL-Light as the following definitions:

Definition 7: ⊢ ∀ P y t. diff_eq_lhs A f t =
vsum (0..LENGTH A) (λ n. Cx (EL n A t) * n_vec_deri n f t)

Definition 8: ⊢ ∀ L y t. diff_eq_rhs L p t =
vsum (0..LENGTH L) (λ n. Cx (EL n L) * EL n p t)

In the above definitions, A and L are the coefficient's lists, f:($\mathbb{R} \rightarrow \mathbb{C}$) and p(t):($\mathbb{R} \rightarrow \mathbb{C}$) are complex-valued functions, and t:($\mathbb{R}$) is the time variable to formally model the linear ordinary differential equation. Definition 6 is also used to formally define the steady-state condition of the power converters as:

Definition 9: $\vdash \forall$ `n. ( steady_state 0 f` T_p `=`
`(n_vec_deri 0 f (&0) = n_vec_deri 0 f` T_p`) )` $\wedge$
`( steady_state (SUC n) f` T_p `=`
`(n_vec_deri (SUC n) f (&0) = n_vec_deri (SUC n) f` T_p`) )`

The above generic formalization allows to formally model the dynamic behavior of systems represented by differential equations. We have utilized this formalization to formally specify and reason the periodic steady-state behavior of power converters, described in Equation 4.

4.3 Solution of Linear Differential Equations

The general solution to non-homogeneous Equation (6) is expressed as

$$y(t) = y_h(t) + y_p(t) = \sum_{i=1}^{n} c_i y_i(t) + y_p(t) \tag{7}$$

Where, $y_h(t)$ is the linear combination of the fundamental solutions of Equation (6) when $p(t) = 0$, and y_p is the particular solution corresponding to Equation (6) when $p(t) \neq 0$.

The formal verification of the correctness of the solution of linear differential equation, i.e., Equation (6), is based on the linearity property of the derivatives, which we have formally verified for the complex-valued functions as:

Theorem 2: $\vdash \forall$ `n f h t.`
A1: `(`λ `m t. m` $\leq$ `n` $\Rightarrow$ `(`λ `t. n_vec_deri m f t) differentiable at t)` $\wedge$
A2: `(`λ `m t. m` $\leq$ `n` $\Rightarrow$ `(`λ`. n_vec_deri m h t) differentiable at t)`
$\Rightarrow$ `n_vec_deri n (`λ`t. Cx a * f t + Cx b * h t) t =`
`Cx a * n_vec_deri (`λ`t. f t) t + Cx b * n_vec_deri (`λ`t. g t) t`

We formally verified the solution of a linear differential equation, represented by Equation (7), in the HOL-Light theorem prover as follows:

Theorem 3: $\vdash \forall$ Y_h `C` Y_p `A L p t.`
A1: `(n_differentiable_fn` Y_h `(LENGTH A))` $\wedge$
A2: `(n_differentiable_fn` Y_p `(LENGTH L))` $\wedge$
A3: `(n_homo_soln A` Y_h `t)` $\wedge$
A4: `(n_nonhomo_soln A L` Y_h Y_p `t)`
$\Rightarrow$ `diff_eq_lhs A (`λ `t. linear_sol C` Y_h `t +` Y_p `t = diff_equ_rhs L p t`

In Theorem 3, Assumptions `A1` and `A2` ensure the n$^{\text{th}}$-order differentiability of the fundamental solutions, given as a list `Yh`, and particular solution, provided as a list `Yp`, respectively. The predicate in the Assumption `A3`, i.e., `n_order_homo_eq_soln_list`, ensures that each element of the list `Yh` is a solution of the given differential equation, when $p(t) = 0$ in Equation (6), where `L` is the list of coefficients. Similarly, the predicate in Assumption `A4`, i.e., `n_order_nonhomo_eq_soln_list`, ensures that the particular solution, `Yp`, satisfies the differential Equation (6). The function `linear_sol`, used in the conclusion of Theorem 2, models the linear solution combination of fundamental solutions, i.e., $\sum_{i=1}^{n} c_i y_i(t)$, using the lists of solution functions `Yh` and arbitrary constants `C`. The formal verification of Theorem 3 is based on Theorem 1 and the formally verified lemma about solution of homogeneous differential equation, i.e., when $p(t) = 0$ in Equation (6). More details about the modeling and verification steps can be found in our proof script [2]. The formalization, presented in this section, is generic and provides sufficient support to formally model and reason about different aspects of a power converters' circuits including; implementation and behavior, specification, correctness of the solution of differential equations representing the behavior of circuits, and also the steady-state behavior of quantities of interests, such as voltages and currents. The corresponding proof script, which is available for download at [2], has 3000 lines of HOL-Light code and requires about 350 man hours of development time.

5 DC-DC Buck Converter

The DC-DC buck converter is a commonly used power converter that steps down a given input to a desired output level. In a DC-DC Buck converter, operating in a continuous conduction mode, a switch controls the flow of energy from the raw source, Vs, to the output by periodically switching between Positions 1 and 2, as shown in Fig 5. The energy is stored in the inductor when the switch is at Position 1, and is dissipated to the output circuitry, when the switch is at Position 2.

The circuit has two modes, i.e., $n = 2$, defined by the switching instances, t_0, t_{on}, and t_{off}. In periodic steady-state the circuit will repeat its behavior periodically over the time period T_p. Moreover, due to periodic steady-state the dependence on t_0 can be dropped and therefore have assigned $t_0 = 0$ in our analysis. Applying Kirchoff's current and voltage laws in switch Positions 1 and 2, gives the following differential equations for the respective modes:

$$\begin{aligned} i_L &= i_C + i_R \\ \frac{d^2}{dt^2}V_{out}^1(t) + \frac{1}{RC}\frac{d}{dt}V_{out}^1(t) + \frac{1}{LC}V_{out}^1(t) &= \frac{V_s}{LC} \\ V_{out}^1(t) &= c_1 e^{s_1 t} + c_2 e^{s_2 t} + V_s \end{aligned} \tag{8}$$

$$\begin{aligned} i_L &= -i_c - i_R \\ \frac{d^2}{dt^2}V_{out}^2(t) + \frac{1}{RC}\frac{d}{dt}V_{out}^2(t) + \frac{1}{LC}V_{out}^2(t) &= 0 \\ V_{out}^2(t) &= c_3 e^{s_3 t} + c_4 e^{s_4 t} \end{aligned} \tag{9}$$

Where, V_{out} is the output voltage of the converter, as shown in the Fig. 5, and s_1, s_2, s_3 and s_4 are the roots of the characteristic equation of the converter in two modes. Moreover, $s_1 = s_3$ and $s_2 = s_4$ due to the identical characteristic equations. The solution of Equations (8-9), over the time period T_c, can be written using the Heaviside step function as

$$V_{out}(t) = u(t - t_{on})V_{out}^1(t) + (1 - u(t - t_{on}))V_{out}^2(t) \tag{10}$$

In the periodic steady-state, the voltage of the DC-DC buck converter satisfies the following conditions

$$V_{out}(0) = V_{out}(T)\,, \;\; \frac{d}{dt}V_{out}(0) \quad = \frac{d}{dt}V_{out}(T) \tag{11}$$

The steady-state conditions provide two algebraic equations, however, there are four constants involved in the solution. Two more algebraic equations can be obtained from the continuity of the voltage, i.e., V_{out}, due to continuous conduction mode of

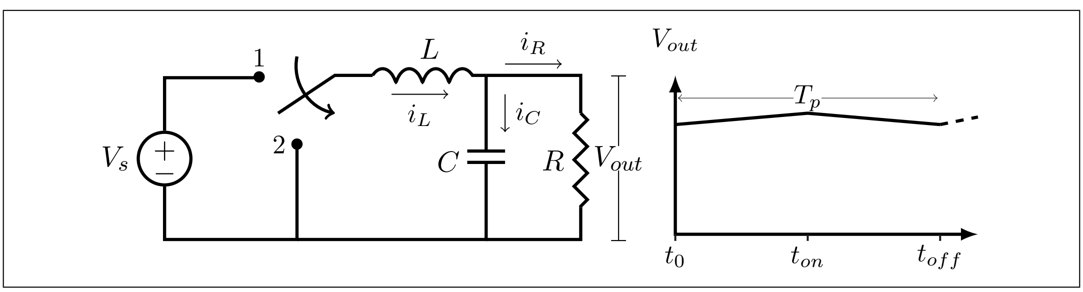

Figure 5: **DC-DC buck Converter**

Component	Current Relationship
Resistor	$\mathtt{I_R(t)} = \frac{\mathtt{V(t)}}{\mathtt{R}}$
Capacitor	$\mathtt{I_C(t)} = \mathtt{C}\frac{\mathtt{dV(t)}}{\mathtt{d(t)}}$
Inductor	$\mathtt{I_L} = \mathtt{i_0} + \frac{1}{\mathtt{R}}\int_0^{\mathtt{t}} \mathtt{V(t)}$

Table 1: **Basic quantities in DC-DC converter**

the circuit, i.e.,

$$V^1_{out}(t_{on}) = V^2_{out}(t_{on})\,,\ \frac{d}{dt}V^1_{out}(t_{on}) \quad = \frac{d}{dt}V^2_{out}(t_{on}) \tag{12}$$

Equations (11-12) are used to specify the periodic steady-state voltage that allows finding the minimum and peak conduction currents in steady-state. These currents can then be used to determine ripple currents, which are essentially crucial in specifying the components in the design of the converters.

The first step, in the formalization of the DC-DC Buck converter consists of using the switching function technique to write the switch junction voltages, which in turn requires to formally define the currents of inductor, capacitor and resistor elements. The mathematical expressions for these elements are presented in Table 1, which are formally defined as,

Definition 10: $\vdash \forall$ `iₒ L v.  ind_curr v L iₒ =`
```
(λ t.  iₒ + Cx (&1 / L) * integral (interval [&0, t]) v)
```

Definition 11: $\vdash \forall$ `C v.  cap_curr C v =`
```
(λ t.  Cx C * vector_derivative v (at t))
```

Definition 12: $\vdash \forall$ `v R. res_curr R v = (λ t.  v t * Cx (&1 / R))`

Where, `R` , `C` and `L` represent the resistance, capacitance and inductances of the resistor, capacitor and inductor of the circuit. $\mathtt{i_o}$ is the initial value of the inductor current, whereas, `v` represents the voltage drop across the circuit elements, at any time `t`. Now, using Definitions 2, 4, 5, 10, 11, and 12, we can formalize the implementation of DC-DC Buck converter as:

Definition 13: $\vdash \forall$ $\mathtt{i_o}$ `L C R` $\mathtt{V_s}$ $\mathtt{V_{out}}$ $\mathtt{V_L}$ $\mathtt{t_{on}}$ `t.`
```
buck_ckt_impl iₒ L C R Vs V_out V_L t_on t =
(Vl = switch_volt [λt.  Cx V_s - V_out t; (λt.  -V_out t)]
     [&1 - semi_switch (t - t_on); semi_switch (t - t_on] t)
```

```
∧ (∀t.  ∼(t = t_on) ⇒
        kcl [ind_curr (λt.  V_L t) L i_o; cap_curr C (λt.  -V_out t);
            res_curr R (λt.  -V_out t)] t )
```

In the above definition, $\mathtt{V_s}$ is the supply voltage, $\mathtt{V_{out}}$ is the voltage drop at the junction of all these components, with respect to the ground, and $\mathtt{V_L}$ is the voltage drop across the inductor. However, due to the the presence of the switching junction, we model the inductor voltage, in the first conjunct, using the `switch_volt` function, which is provided with two lists; one for all the possible voltage drops, and the other with all the corresponding switching functions for every mode, and an independent variable `t`. Where, $\mathtt{t_{on}}$, is the exact switching instant. This voltage is then used to apply the conventional Kirchoff's current law, using the function `kcl`, which accepts a list of currents, and an independent variable, i.e., `t`.

This implementation model results in the ordinary linear differential equations of the system, which can be described using Definitions 7 and 8 as:

Definition 14: $\vdash \forall\ \mathtt{i_o\ V_s\ V_{out}\ L\ C\ R\ t_{on}\ t}.$

$\mathtt{buck\_diff\_equ\ i_o\ V_s\ V_{out}\ L\ C\ R\ t_{on}\ t} =$

$\mathtt{if\ (t < t_{on})\ then\ diff\_eq\_lhs\ [\frac{1}{LC}; \frac{1}{RC};\ 1]\ (V_{out}(t))\ t} =$

$\mathtt{diff\_eq\_rhs\ [\frac{V_s}{LC}]\ [1]\ t}$

$\mathtt{else\ diff\_eq\_lhs\ [\frac{1}{LC}; \frac{1}{RC};\ 1]\ (V_{out}(t))\ t = diff\_eq\_rhs\ [0]\ [0]\ t}$

According to the proposed methodology, as a first step, we formally verify the implementation and behavior of the Buck converter using the formal model of switching function technique and linear order differential equations as:

Theorem 4: $\vdash \forall\ \mathtt{i_0\ V_s\ V_L\ V_{out}\ L\ C\ R\ t_{on}\ T_p\ t}\ .$

A1: $(\forall\ \mathtt{t}.\ \mathtt{V_L\ continuous\_on}\ [0, \mathtt{t}]\ \wedge$

A2: $\sim\ (\mathtt{C} = 0)\ \wedge$

A3: $(\mathtt{t} \in (0,\ \mathtt{T_p}))\ \wedge$

A4: $\sim(\mathtt{t} = \mathtt{t_{on}})\ \wedge$ **A5:** $(\mathtt{t_{on}} \in (0,\ \mathtt{T_p}))\ \wedge$

A6: $(\forall\ \mathtt{t}.\ \mathtt{differentiable\_n\_vec\_deri}\ 1\ \mathtt{V_{out}\ t})\ \wedge$

A7: $\mathtt{buck\_ckt\_impl\ i_0\ L\ C\ R\ V_s\ V_{out}\ V_L\ t_{on}\ t}$

$\Rightarrow \mathtt{buck\_diff\_equ\ i_0\ V_s\ V_{out}\ L\ C\ R\ t_{on}\ t}$

Assumption `A1` ensures that the converter is operating in the continuous conduction mode. Assumption `A2` prevents a division by zero case in the formal analysis. Assumptions `A3`-`A4` ensure that the time is over one time period of the system and does not include the singularities, at $\mathtt{t_0} = 0$, $\mathtt{t} = \mathtt{t_{on}}$ and $\mathtt{t} = \mathtt{T_p}$, due to switching action. Whereas, Assumptions `A5` specifies that the switching time, $\mathtt{t} = \mathtt{t_{on}}$, lies within the open interval defined by the single time period of the circuit. Assumption

A6 formally specifies the differentiability of the function, V_{out}, and its first derivative. The predicate differentiable_n_vec_deri accepts a number, n, and function, f, and specifies the differentiability of the function upto its n^{th}-derivative. Finally, Assumption A7 specifies the formal implementation of the power converter circuit using Definition 13. The formal proof of Theorem 4 involves taking derivative of Assumption A7, which consists of piecewise functions, by employing Theorem 1.

Following the proposed methodology, the next task is to formally verify the correctness of the solution of the ordinary linear differential equations of the Buck converter in HOL-Light. Therefore, we define the piecewise solution, i.e., Equation (10), of the Buck converter in higher-order logic as:

Definition 15: $\vdash \forall\ V_s\ c_1\ c_2\ c_3\ c_4\ s_1\ s_2\ t_{on}\ t.$

```
solution Vs c1 c2 c3 c4 s1 s2 ton t =
linear_sol [c1; c2] (cexp_list [s1; s2]) t *
           Cx (semi_switch (t - ton)) +
linear_sol [c3; c4] (cexp_list [s1; s2]) t *
          Cx (&1 - semi_switch (t - ton)
```

Where V_s is the supply voltage, c_1, c_2, c_3 and c_4 are arbitrary constants, s_1 and s_2 are the roots of homogeneous differential equations corresponding to Equations (7) and (8), respectively. Whereas, the cexp_list function is a higher-order-logic function to express the exponential form of the solution for real and distinct roots, i.e., s_1 and s_2, of the circuit. It is defined as:

Definition 16: $\vdash \forall$ x. (cexp_list [] = []) $\wedge$

```
cexp_list (CONS s t) = CONS (λx. cexp (s * Cx (x))) (cexp_list t)
```

Next, using Definition 15, we formally verify the correctness of the solution of the differential equations, in each mode of the converter, in HOL-Light as:

Theorem 5: $\vdash \forall\ i_0\ V_s\ V_{out}\ L\ C\ R\ c_1\ c_2\ c_3\ c_4\ s_1\ s_2\ t_{on}\ T_p\ t\ .$

A1: $(\forall\ t.\ \sim(t = t_{on}) \Rightarrow V_{out} = \text{solution}\ V_s\ c_1\ c_2\ c_3\ c_4\ s_1\ s_2\ t_{on}\ t) \wedge$

A2: $(s_1 = -\frac{1}{2RC} + \frac{1}{2}\sqrt{\frac{1}{(RC)^2} - \frac{4}{LC}}\ ) \wedge$

A3: $(s_2 = -\frac{1}{2RC} - \frac{1}{2}\sqrt{\frac{1}{(RC)^2} - \frac{4}{LC}}\ ) \wedge$

A4: $(4\ R^2\ C \leq L) \wedge$

A5: $(0 < L) \wedge$

A6: $(0 < R) \wedge$

A7: $(0 < C) \wedge$

A8: $(t \in (0,\ T_p)) \wedge$

A9: $\sim(t = t_{on}) \wedge$

A10: $(t_{on} \in (0,\ T_p))$

$\Rightarrow$ buck_diff_equ $i_0\ V_s\ V_{out}\ L\ C\ R\ t_{on}\ t$

Assumption A1 formally defines the output voltage $\mathtt{V_{out}}$ as a piecewise function, over the time period, $\mathtt{T_p}$, of the converter circuit. Assumptions A2-A3 formally specify the roots of the equation. Assumption A4 formally specifies the condition on the circuit parameters for real and distinct roots. Assumptions A5-A7, ensure the positive values of inductance, resistance and capacitance of the circuit. Assumptions A8-A9 ensure that the time is over one time period of the system and does not include the singularities, at $\mathtt{t_0} = \mathtt{0}$, $\mathtt{t} = \mathtt{t_{on}}$ and $\mathtt{t} = \mathtt{T_p}$, due to switching action. Whereas, Assumptions A10 specifies that the switching time, $\mathtt{t} = \mathtt{t_{on}}$, lies within the open interval defined by the single time period of the circuit.

The formal verification of Theorem 5 utilized the formally verified results of Theorems 1 and 3.

Finally, we present the formally verified results of periodic steady-state voltage of of the DC-DC Buck converter as:

Theorem 6: $\vdash \forall\ \mathtt{V_s\ V_{out}\ c_1\ c_2\ c_3\ c_4\ s_1\ s_2\ t_{on}\ t\ T_p}.$

A1: $(\mathtt{t} \in (\mathtt{0},\ \mathtt{T_p})) \quad \wedge$

A2: $\sim(\mathtt{t} = \mathtt{t_{on}}) \quad \wedge$

A3: $(\mathtt{t_{on}} \in (\mathtt{0},\ \mathtt{T_p}))\ \wedge$

A4: $(\forall\ \mathtt{t}.\ \sim(\mathtt{t} = \mathtt{t_{on}}) \Rightarrow \mathtt{V_{out}} = \mathtt{solution\ V_s\ c_1\ c_2\ c_3\ c_4\ s_1\ s_2\ t_{on}\ t})\ \wedge$

A5: $(\forall\ \mathtt{t}.\ \ \mathtt{n\_vec\_deri\ 1}\ (\lambda\ \mathtt{t}.\ \ \mathtt{V_{out}\ t})\ \mathtt{continuous\ at\ t})\ \wedge$

A6: $\sim(\ \mathtt{s_2} - \mathtt{s_1} = \mathtt{0}) \quad \wedge$

A7: $\mathtt{steady\_state\ 1\ V_{out}\ t} \Rightarrow$

$$\Bigg(\mathtt{V_{out}}(0) = \left(\frac{\mathtt{s_2}}{\mathtt{s_2} - \mathtt{s_1}}\right)\Bigg[\ \Bigg(\ \mathtt{V_{out}}(0) + \frac{1}{\mathtt{s_2}}\frac{d}{dt}\mathtt{V_{out}}(0) - \mathtt{V_s}\Bigg)\ e^{-\mathtt{t_{on}}\mathtt{s_1}} + \mathtt{V_s}\Bigg]\ e^{-\mathtt{T_p}\mathtt{s_1}} +$$
$$\left(\frac{\mathtt{s_1}}{\mathtt{s_2} - \mathtt{s_1}}\right)\Bigg[\ \Bigg(\ -\mathtt{V_{out}}(0) - \frac{1}{\mathtt{s_1}}\frac{d}{dt}\mathtt{V_{out}}(0) + \mathtt{V_s}\Bigg)\ e^{-\mathtt{t_{on}}\mathtt{s_1}} - \mathtt{V_s}\Bigg]\ e^{-\mathtt{T_p}\mathtt{s_2}}\Bigg)\ \wedge$$
$$\Bigg(-\frac{d}{dt}\mathtt{V_{out}}(0) = \left(\frac{\mathtt{s_1}\mathtt{s_2}}{\mathtt{s_2} - \mathtt{s_1}}\right)\Bigg[\ \Bigg(\ \mathtt{V_{out}}(0) + \frac{1}{\mathtt{s_2}}\frac{d}{dt}\mathtt{V_{out}}(0) - \mathtt{V_s}\Bigg)\ e^{-\mathtt{t_{on}}\mathtt{s_1}} + \mathtt{V_s}\Bigg]$$
$$e^{-\mathtt{T_p}\mathtt{s_1}} + \left(\frac{\mathtt{s_1}\mathtt{s_2}}{\mathtt{s_2} - \mathtt{s_1}}\right)\Bigg[\ \Bigg(\ -\mathtt{V_{out}}(0) - \frac{1}{\mathtt{s_1}}\frac{d}{dt}\mathtt{V_{out}}(0) + \mathtt{V_s}\Bigg)\ e^{-\mathtt{t_{on}}\mathtt{s_1}} - \mathtt{V_s}\Bigg]\ e^{-\mathtt{T_p}\mathtt{s_2}}\Bigg)$$

Assumptions A1 and A2 formally specify the analysis over one time period with singularities, at $\mathtt{t} = \mathtt{0}$, $\mathtt{t} = \mathtt{t_{on}}$ and $\mathtt{t} = \mathtt{T_p}$, excluded. Whereas, Assumptions A3 specifies that the switching time, $\mathtt{t} = \mathtt{t_{on}}$, lies within the open interval defined by the single time period of the circuit. Assumption A4 formally defines the output voltage $\mathtt{V_{out}}$ as a piecewise function, over the time period, $\mathtt{T_p}$, of the converter circuit. Assumption A5 formally specifies the continuity of the function and its derivative, to ensure the continuous conduction mode. Assumption A6 prevents the division by zero case in the analysis, and finally, Assumption A7 defines the steady-state of the buck converter.

The formal proof of Theorem 6 essentially consists of finding the values of the function and its derivative at $\mathtt{t} = \mathtt{0}$ and $\mathtt{t} = \mathtt{T_p}$, in limit sense, and the values of arbitrary constants c_1, c_2, c_3 and c_4 by utilizing the continuity assumption A5 and the one-sided limits concepts due to singularities at $t = 0$, $t = t_{on}$ and $t = T_p$, due to switching action. More details about the proof can be found at [2].

The proposed foundational formalization of switching function technique and linear differential equations allowed us to formally specify and verify the nonlinear behavior of the DC-DC Buck converters in a very straightforward manner. Theorem 4 verifies that the implementation and behavior of the Buck converter by explicitly specifying the conditions on the piecewise functions, e.g., voltages in the case of DC-DC Buck converter, in the continuous conduction operating mode of the converter. The formally verified result is very helpful in the topology selection of the converter, which is usually the first step in the design procedure and, in practice, consists of an intuitive selection of topology for a given design specification. Moreover, Theorem 5 formally verifies the correction of the solution of the linear order differential equations representing the power converter behavior. This result plays a vital role in the performance evaluation. Once the implementation and behavior (Theorem 4), and the solution (Theorem 5) of the DC-DC Buck converter is formally verified, then Theorem 6 formally verifies the relationship among different parameters of the circuit, such as voltage and circuit components, in periodic steady-state. This result is instrumental in formal verification of the design objectives, such as desired voltage levels and component values, of the circuit. However, unlike traditional techniques these formally verified results give exact conditions in terms of the parameters of the Buck converter as they have been formally verified using a sound theorem prover. Moreover, these results are generic in terms of universally quantified variables and contain an exhaustive set of assumptions required for the validity of the results.

6 Conclusion

In this paper, we presented a formal methodology to conduct the formal time-domain based periodic steady-state analysis of power converters. The power converters are characterized by the switching functionality, which imparts to the structural changes of the converter circuit and a nonlinear mathematical analysis. To model the structural changes in the circuit, we developed the formal model of the circuit analysis technique, called switching function technique, and also developed a formal model of linear differential equations to formally specify the behavior of the converters. To cater for the nonlinearities in the analysis, the integral property of the Heaviside step function as a generalized function is verified. This logical formalism is then applied to the DC-DC Buck converter to formally verify the implementation and behavior of the converter's circuit, solution of its linear ordinary differential equations in all modes of the converter's circuit and the steady-state voltage relationship of the DC-DC Buck converter.

The proposed formalization can be extended to incorporate the formal small-signal modeling analysis of the power converters. Moreover, the formalization is based upon the complex valued functions to formally analyze the periodic steady-state analysis of power converters, which are characterized by the discontinuity due to switching action, therefore, the formalization is also equally applicable to analyze many other discontinuous phenomenon ubiquitous in many fields of Physics and engineering.

References

[1] Milton Abramowitz and Irene A. Stegun. *Handbook of mathematical functions: with formulas, graphs, and mathematical tables*, volume 55. Courier Corporation, 1964.

[2] Asad Ahmed. Formal periodic steady-state analysis of power converters in time-domain. `http://save.seecs.nust.edu.pk/projects/fpssapc/`. [Online; accessed 1-March-2019].

[3] Achraf Ben Amar, Ammar B. Kouki, and Hung Cao. Power approaches for implantable medical devices. *Sensors*, 15(11):28889–28914, 2015.

[4] B. Jayant Baliga. *Fundamentals of power semiconductor devices.* Springer Science & Business Media, 2010.

[5] Soumitro Banerjee and George C. Verghese. *Nonlinear phenomena in power electronics.* Wiley-IEEE Press, 2001.

[6] Sidi Mohamed Beillahi, Umair Siddique, and Sofiène Tahar. Formal analysis of power electronic systems. In *International Conference on Formal Engineering Methods*, pages 270–286. Springer, 2015.

[7] Philip J. Davis and Philip Rabinowitz. *Methods of numerical integration.* Courier Corporation, 2007.

[8] Manjusha Dawande, Victor Donescu, Ziwen Yao, and V. Rajagopalan. Recent advances in simulation of power electronics converter systems. *Sadhana*, 22(6):689–704, 1997.

[9] Ali Emadi. *Handbook of automotive power electronics and motor drives.* CRC Press, 2017.

[10] Robert W. Erickson and Dragan Maksimovic. *Fundamentals of power electronics.* Springer Science & Business Media, 2007.

[11] John Harrison. HOL Light: An overview. In *International Conference on Theorem Proving in Higher Order Logics*, pages 60–66. Springer, 2009.

[12] John Harrison. The HOL Light theory of Euclidean space. *Journal of Automated Reasoning*, 50(2):173–190, 2013.

[13] M. David Kankam and Malik E. Elbuluk. A survey of power electronics applications in aerospace technologies. `https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.`

gov/20020013943.pdf, 2001. [Online; accessed 1-March-2019].

[14] Ram P. Kanwal. *Generalized functions: Theory and technique.* Springer Science & Business Media, 2012.

[15] Tuo Yeong Lee. *Henstock-Kurzweil integration on Euclidean spaces*, volume 12. World Scientific, 2011.

[16] Dragan Maksimovic. Automated steady-state analysis of switching power converters using a general-purpose simulation tool. In *Power Electronics Specialists Conference*, volume 2, pages 1352–1358. IEEE, 1997.

[17] Christos C. Marouchos. *The switching function: Analysis of power electronic circuits*, volume 17. The Institution of Engineering and Technology(IET), 2006.

[18] Marcia Verônica Costa Miranda and Antônio Marcus Nogueira Lima. Formal verification and controller redesign of power electronic converters. In *Industrial Electronics, IEEE International Symposium on*, volume 2, pages 907–912, May 2004.

[19] Muhammad Usman Sanwal and Osman Hasan. Formally analyzing continuous aspects of cyber-physical systems modeled by homogeneous linear differential equations. In *International Workshop on Design, Modeling, and Evaluation of Cyber Physical Systems*, pages 132–146. Springer, 2015.

[20] Matthew Senesky, Gabriel Eirea, and Tak-John Koo. Hybrid modelling and control of power electronics. In *International Workshop on Hybrid Systems: Computation and Control*, pages 450–465. Springer, 2003.

[21] David R. Stoutemyer. Crimes and misdemeanors in the computer algebra trade. *Notices of the American Mathematical Society*, 38(7):778–785, 1991.

[22] Thomas G. Wilson. Life after the schematic: The impact of circuit operation on the physical realization of electronic power supplies. *Proceedings of the IEEE*, 76(4):325–334, 1988.

Received 13 August 2019