

# Formal Reliability Analysis of Protective Relays in Power Distribution Systems

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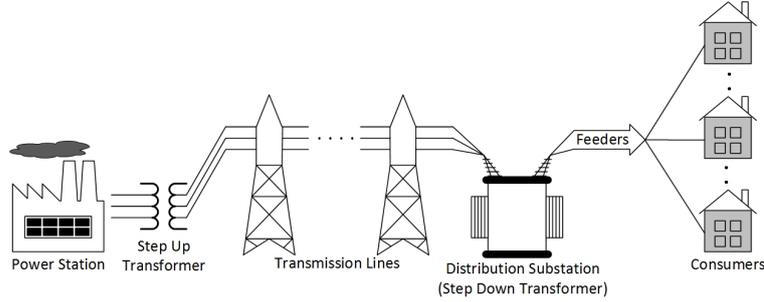
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**Abstract.** Relays are widely used in power distribution systems to isolate their faulty components and thus avoid disruption of power and damaging expensive equipment. The reliability of relay-based protection of power distribution systems is of utmost importance and is judged by first constructing Markovian models of individual modules and then analyzing these models analytically or using simulation. However, due to their inherent limitations, simulations and analytical methods cannot ascertain accurate results and are not scalable, respectively. To overcome these limitations, we propose a modular approach for developing Markovian models of relay-based protected components and then analyzing the reliability of the overall power distribution system by executing its individual modules in parallel using the PRISM probabilistic model checker. The paper presents a foundational model for a relay-based protected component that can be incrementally updated to represent more advanced behaviors, such as self-checking, routine test and continuous monitoring. Moreover, the paper provides a set of reliability assessment properties of power distribution systems that can be formally verified by PRISM. For illustration purposes, we present the analysis of a typical power distribution substation.

**Keywords:** Probabilistic Model Checking, PRISM, Markov Chains, Relay-based Protection Systems

## 1 Introduction

Power distribution systems [6], depicted in Figure 1, are used to ensure reliable distribution of electricity, generated by power stations, to end users. The main idea behind this enormously used distribution network is to first route power from the main power station to various substations via transmission lines. The substations then in turn distribute the power to their respective consumers. Besides the transmission lines, the other integral component of power distribution networks is a power transformer. It is installed at the main power station to step up the generation level voltages (11kV or 33kV) to transmission levels (230kV or 69kV) or at the substations to step down the transmission level voltages to distribution levels (34.5kV or 24.kV) [24].



**Fig. 1.** A Typical Power Distribution System

The power distribution system is a highly sensitive and safety-critical domain. Faults like switching surges and short circuits can not only damage the sophisticated and expensive components, like transmission lines or transformers, of a power distribution system but could also lead to catastrophic consequences like the major power failure in the UCTE grid [13] or the 2005 Moscow power blackout [11]. In order to protect the power distribution systems and their components, it has now become customary to associate a protective relay [21] with every safety-critical component of a power distribution system. These protective relays are capable of sensing the fault and then isolating the faulty component from the power distribution system by tripping the circuit breaker. The protective relays have undergone a transformation from electromechanical components to presently deployed multi-functional digital relays.

The reliability of the relay-based protection system of power distribution is of utmost importance. The main idea behind the reliability analysis of these protection systems is to model the behavior of the given system, along with its unpredictable elements, like fault occurrence, as a Markov chain and then analyze this model to find probabilities associated with various parameters of interest, like failures, testing intervals, fault clearing times etc. Traditionally, this reliability analysis of power distribution systems is done either analytically or using computer-based simulation. In the analytical approach, a Markovian model of the given relay-based protection system is developed on paper and the desired properties in this model are either judged analytically using paper-and-pencil proofs or computer based numerical methods. Simulation technique, on the other hand, considers the stochastic behavior of the given system while treating it as a series of real experiments [5]. Both of the above mentioned techniques have their shortcomings. Analytical techniques are more accurate compared to the simulation techniques, which compromise the completeness of the analysis by considering only a subset of all possible scenarios. On the other hand, the analytical approaches are not scalable and simulation allows us to analyze more complex and larger systems. Moreover, the absolute accuracy of analytical analysis cannot be guaranteed as it is prone to human errors and the rounding errors of numerical methods. Another major drawback of the existing

reliability analysis methods for the relay-based protection systems is that the Markovian model of the given system is usually developed on paper and in an ad-hoc manner. This kind of an informal modeling approach has limited scalability and is also quite prone to human errors. Due to the above mentioned scalability issues, all the existing reliability analyses of relay-based protection systems focus on the reliability assessment of single components rather than the complete power distribution system. Whereas, the inaccuracy limitations have been reported as the main causes behind the 2003 Northeast blackout in the United States and Canada [12] and the 2012 blackout in India [4].

Formal methods [2] are capable of overcoming the above mentioned inaccuracy limitation and have been successfully used to guarantee correctness of many real-world software, hardware and physical systems. However, to the best of our knowledge, no prior work regarding the formal reliability analysis of power distribution systems exists so far. In order to fill this gap, we propose to use probabilistic model checking [18], which is a widely used formal method for analyzing Markovian models. In particular, the paper presents a generic and formal approach to analyze the reliability of power distribution systems that use digital relay based protection. We provide a foundational Markovian model for a power distribution component that is protected by a digital relay. This proposed model can be used to represent the behavior of any relay protected component, such as transmission line or transformer, of a power distribution system. The main distinguishing feature of this model is that it allows the incorporation of additional states and transitions, to build more complex and advanced relay models, in a methodological way. The Markovian models of different relay-based protected components can then be combined to model and analyze the reliability of the complete power distribution system using a probabilistic model checker.

The proposed approach provides more accurate results than the traditional counterparts due to the exploration of an exhaustive state-based model of the given power distribution system. We have chosen the PRISM model checker [20] for the proposed work as it supports CSL and can analyze larger models in a very efficient manner. It also supports the evaluation of expected values, specified in terms of reward functions, which allows us to verify some interesting properties in the context of relay-based protection for power distribution systems. In order to illustrate the usefulness of the proposed approach, we utilize it to analyze the reliability of a typical power distribution subsystem [24]. To the best of our knowledge, this is the first reliability analysis of a complete power distribution system that caters for simultaneous failures of multiple components.

The paper is organized as follows. A brief review of the available research regarding reliability analysis of digital relays and probabilistic model checking is presented in Section 2. Section 3 describes our foundational protective relay model and the proposed approach for its incremental refinements. Section 4 provides an overview of the PRISM model checker along with the proposed properties of interest in the context of reliability analysis of power distribution systems. Section 5 presents the case study along with the verification results and some discussions. Finally, Section 6 concludes the paper.

## 2 Related Work

Grimes [14] laid the foundations for the reliability assessment of electric power systems by presenting mathematical equations for the failure probabilities of protective relays. A simple protection system reliability model is developed in [23] that establishes a relationship between the un-readiness probability and the undetected failure rate. This model is then further refined in [3] by including back-up protection, common cause failure and fault clearing phenomena. In [22], a Markovian model for the transmission line is proposed for determining the optimum routine test and self-checking intervals to maximize the reliability. This model is extended in [16] by including the parameters of inrush currents and over excitation to analyze the protective system of a power transformer. Both of these models considered the backup relay to be fully reliable. This limitation is overcome in [7] by considering the failure probability associated with the backup relay. Another model presented in [10] takes into consideration the causes of relay failures including software, hardware, auxiliary equipment and human errors. This model also investigates routine test effectiveness, level of reliance on self-checking, stuck breakers and human errors during tests and repair actions. In [1], the routine test approach is further improved by considering the impacts of individual components of the protective relaying system, such as current transformer (CT), voltage transformer (VT), Power Supply Unit (PSU), Circuit Breaker (C.B) and the trip coil. An optimum routine test inspection interval is then determined for each component individually and its comparison with the conventional method is given. All of the above mentioned models have been developed in an ad-hoc way, i.e., without following any particular principles or rules, and they are analyzed using numerical methods and simulations, which compromise the accuracy of reliability assessment. Moreover, due to the non-scalable modeling approach and the high computation requirements of numerical methods, only the models of single components have been analyzed. In this paper, we alleviate these problems by proposing a scalable modeling approach and probabilistic model checking based verification.

Many probabilistic model checkers, including PRISM [20], YMER [25], MRMC [17], VESTA [19] and ETMCC [15] are available. The main objective of the proposed work is to find steady state probabilities of power distribution systems, which are usually modeled as continuous time Markov chains (CTMC). However, YMER and VESTA do not support steady state probabilities and thus cannot be used for our purpose. The PRISM model checker has been reported as the most efficient one in terms of memory consumption [9] compared to MRMC and ETMCC. Thus, we have used the PRISM model checker for this work as it also supports analyzing CTMCs and offers a very user friendly graphical interface.

## 3 Modeling Relay-based Protected Components

The proposed modeling approach is primarily inspired from Endrenyi's three-state Markovian model [9] used to analyze the behavior of a component under

fault. According to this model, the component can be in one of the three possible states; normal operating state, failed state or isolated state. When the fault occurs, the model moves from the normal operating state to the failed state. Relay, protecting the component, isolates it from the system thereby undergoing a transition to the isolated state. The isolated component is then restored to the normal operating state after repair.

The proposed foundational model of a relay-based protected component (Figure 2) caters for temporary and permanent component faults and the faults that may appear in the relay themselves. Temporary faults exist only for a short period of time and the circuit can be re-energized without repairing the component. Permanent faults, on the other hand, damage the component permanently and component repair action is necessary for their clearance. The above mentioned three-state model can be used to model both temporary and permanent faults. The difference being the transition rates from the faulty to normal state, i.e., the model returns to the normal state with reclosing switching rate ( $\lambda_{rct}$ ) in the case of a temporary fault and the component repair rate ( $\mu_c$ ) in the case of a permanent fault. Two types of relay failures, i.e., internal and external are

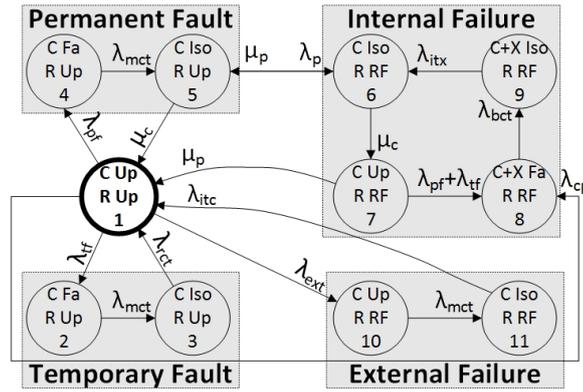


Fig. 2. Foundational Model of a Relay-based Protected Component<sup>1</sup>

considered in the proposed model. Relay failures due to external faults cause the model to transition to state 10 with rate  $\lambda_{ext}$ . The component is then isolated in state 11, with the main switching rate of the component ( $\lambda_{mct}$ ). The component is restored with the switching rate  $\lambda_{itc}$  when the external fault in the relay is rectified. Relay failures due to internal relay faults can also occur with a failure rate  $\lambda_p$ , when the component has been isolated with an identified permanent fault, i.e., state 5, thereby causing a transition to state 6. In this

<sup>1</sup> C UP = component is energized, C Fa = component is faulty, C Iso = component is isolated, R UP = relay is operational, R RF = revealed failure, X = additional equipment

state, the relay has been identified as faulty and the component is isolated due to a permanent fault, and therefore, either of them can be repaired first. If the relay is repaired first, the model moves back to state 5 with the relay repair rate ( $\mu_p$ ). The component is then repaired, with component repair rate ( $\mu_c$ ), and restored to the normal operating conditions (state 1). On the other hand, if the component is repaired first ( $\mu_c$ ), then the model transitions to state 7, after which, relay repairing action ( $\mu_p$ ) moves the model back to state 1. While in state 7, if either type of component fault occurs, the model transitions to state 8 in which the component, the relay and the additional equipment (X) connected to the component are all considered faulty. The common cause failure rate of the component and the relay is  $\lambda_{cp}$  and the model transitions from its normal state (state 1) to state 8 with this rate. It is assumed that the back-up relay is fully reliable and its switching operation with rate  $\lambda_{bct}$  isolates both the equipment (X) and the component (state 9). The additional equipment (X) is first restored with switching rate  $\lambda_{itx}$  in state 6, followed by the component repair action and restoration of the component as mentioned above.

The proposed model of Figure 2 is very generic and can be extended to represent more advanced behaviors of relay-based protected components. For example, the testing procedures for self-checking, routine test and continuous monitoring, can be added to the basic model mentioned above. During the self-checking test, the relay checks for the correct operation of its core components and is unavailable for normal operation. Similarly, the relay also becomes unavailable for operation in the event of routine test inspection during which a detailed analysis of operation of the digital relay is performed. The relay is available for service only during continuous monitoring in which only its critical components are checked for correctness. Routine tests and self-checking can detect relay failures and mal-trips and take appropriate actions to remove these faults. Mal-trips can be either instantaneous, i.e., they manifest immediately, or potential, which require certain conditions to manifest.

In order to illustrate the generic nature of the proposed foundational model of Figure 2, we form the model of a relay-based protected component that supports the routine test capability in Figure 3. This can be mainly done by including some more states and transitions in the model of Figure 2. The foundational model of Figure 2 is represented by the grey shaded region in Figure 3 and for the sake of simplicity, only the states that interact with the new states are mentioned. The model moves to state 12 when routine tests are being carried out and relay is unavailable for operation with rate  $\lambda_{rt}$ , which represents the frequency with which the routine test procedures are carried out. At this state, permanent faults (state 15) in the component and instantaneous mal-trips (state 8) can occur with relay failure rate ( $\lambda_p$ ) and instantaneous mal-trip rate ( $\lambda_{rt-op}$ ), respectively. When in state 15, the model transitions to state 16 with the manual switching rate ( $\lambda_{mct}$ ) where the component is isolated. The component is then re-energized manually with rate  $\lambda_{itc}$  in state 9 after which the system returns to its normal operating conditions once the repairing of the relay is complete ( $\mu_p$ ). States 13 and 14 represent the portion of relay failures due to internal

faults ( $\lambda_{prt}$ ) and potential mal-trip ( $\lambda_{pt-rt}$ ), respectively, that are detectable by routine tests. These faults, when detected by routine tests, transfer the model to state 9 with rate  $\lambda_{rt}$ . Consequently, the relay is repaired as mentioned in the basic model description. It is also possible that before the detection of relay failure, due to potential mal-trip or internal relay failure by routine test, the component gets a permanent fault ( $\lambda_{pf}$ ), which will lead to the isolation of the corresponding component and additional equipment (X).

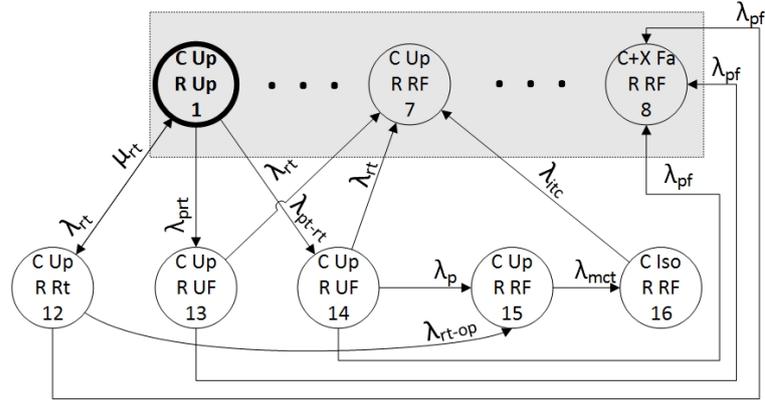


Fig. 3. Basic Model with Routine Test<sup>2</sup>

Similarly, the proposed generic model, given in Figure 2, can be used to construct reliability models with the self checking capability [22,16]. In this case, the instantaneous mal-trips cannot occur while the relay is under self-checking. This can be done by removing the transition from state 12 to 15 and replacing the transition rates with the corresponding ones for self-checking in the model of Figure 3. Similarly, continuous monitoring [22] can be included in the model by adding a transition from normal operating state to state 7 with rate  $\lambda_{pmn}$ <sup>3</sup>.

The existing models (e.g. [22,16,1]) for the reliability analysis of protective relays are specific for one particular scenario and, to the best of our knowledge, no fixed set of rules is available to develop or incrementally update these models. Thus, the modularity and the ability for incremental updates in the behavior are the main distinguishing features of the proposed modeling approach. This feature facilitates the construction of system-level models of power distribution systems by allowing reusability of components and also is less error prone due to the modular and regular development process.

<sup>2</sup> R UF = unrevealed failure, R Rt = relay under routine test inspection

<sup>3</sup>  $\lambda_{pmn}$  = portion of relay failure detected by continuous monitoring

## 4 Verification of Reliability Properties

We propose to analyze the reliability of the relay-based protective system of a complete power distribution substation by executing the Markovian models of its individual components in parallel. The PRISM model checker supports for parallel execution of different processes through parallel composition of its modules and is thus used for modeling and analyzing complex power distribution systems in this paper. The main idea of the proposed approach is to first describe the Markovian models of individual modules, such as transmission line or transformer, of the power distribution system, based on the foundational model of Figure 2, as a PRISM module. These individual modules are then combined in a single PRISM model for their parallel execution. This way, we analyze the reliability of the real-world scenario, where multiple faults can simultaneously occur in various components of a power distribution system. To the best of our knowledge, this kind of an overall power distribution system analysis has not been reported in the open literature so far.

In this section, we first provide a quick overview of the PRISM model checker. This is followed by the description of the properties that we propose to verify using PRISM for the reliability assessment of relay-based protection systems.

### 4.1 The PRISM Model Checker

PRISM is a probabilistic model checker for the construction and analysis of Markov Chains, Markov Decision Processes (MDPs) and Probabilistic Timed Automatas (PTAs). The models are described using a state-based language called the PRISM language. Modules and variables are basic components of the modeling language. A model can consist of a number of modules whose state at a given time is represented by the values of local variables defined in those modules. The values of local variables of all the modules define the overall state of the system. A set of guarded commands describe the behavior of each module:

$$[] \textit{guard} \rightarrow \textit{prob}_1 : \textit{update}_1 + \textit{prob}_2 : \textit{update}_2 \dots + \textit{prob}_n : \textit{update}_n;$$

The *guard* is a predicate over all the variables and a command is enabled when its guard becomes true. Each *update<sub>i</sub>* defines a possible transition with probability *prob<sub>i</sub>*. PRISM provides support for a variety of property specifications such as PCTL, CSL, LTL and PCTL\*. For example:

$$S_{\geq 0.99}[\textit{“normal”}] - \textit{“is the steady state probability of normal state} \geq 0.99\textit{”}$$

PRISM supports verification and analysis of time based properties which we use for the time based analysis of Markovian models. These properties are analyzed by associating a certain reward with each state of the model through a reward structure. PRISM also allows the use of customized properties using the *filter* operator: *filter*(*op*, *prop*, *states*), where *op* represents the filter operator (min, avg, max), *prop* represents the PRISM property and *states* (optional field) represents the set of states over which to apply the filter.

## 4.2 Reliability Properties

The purpose of reliability analysis of protective relays deployed in power distribution systems is to minimize the steady-state probabilities of undesirable states and maximize the ones of desirable states. This is achieved by optimizing the values of different testing intervals. The classification between desirable and undesirable states is based on labeling the model states as *normal*, *dependability*, *security* and *unavailability* states where *normal* and *dependability* are the desirable states and *security* and *unavailability* are the undesirable states [8].

In the *normal* state of the model, both the component and the relay are operational with no faults. Thus, State 1 of Figure 3, can conveniently be labeled as the *normal* state. *Dependability* is the probability associated with the correct operation of the relay, i.e., operating when required. Therefore, the states 2, 3, 4, and 5 in Figure 3 can be classified as the *dependability* states because in these states, the relay has operated due to a component fault. *Security*, on the other hand, is the probability of an unnecessary operation of the relay. Undesired operation of the relay occurs due to mal-trip of the relay and thus the states 10, 11, 15 and 16 in Figure 3 are the *security* states. The states in which the relay is either faulty or unavailable due to testing procedures (self-checking or routine test) are labeled as the *unavailability* states because of the fact that the relay is unavailable for operation. The states 6, 7, 8, and 9 of Figure 3 fall into the category of *unavailability* states.

The steady state probabilities of a component model can now be determined using the labels defined above. For example, the steady state probability of *unavailability* state can be obtained by verifying the following property in PRISM:

$$S_{=?} [ \text{"unavailability"} ] - \text{"steady state probability of unavailability state"}$$

where, in the case of Figure 3, the *unavailability* would represent the states 6, 7, 8 and 9. When determining the steady state probabilities at the system level, the AND(&) operator is used for *normal* and *dependability* states. For example for the system to be in the healthy condition, all of its component models must be in their respective *normal* states at the same time:

$$S_{=?} [ \text{"normal}_1" \ \& \ \text{"normal}_2" \ \& \ \dots \ \text{"normal}_i" ]$$

On the other hand, the steady state probabilities of *security* and *unavailability* states are determined using the OR(|) operator because even a single component fault can compromise the security of the whole system:

$$S_{=?} [ \text{"unavailability}_1" \ | \ \text{"unavailability}_2" \ | \ \dots \ \text{"unavailability}_i" ]$$

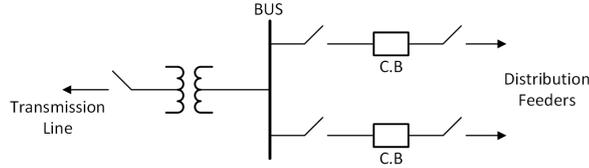
Time based properties, such as "fault clearing time", can be analyzed using the reward/cost structures. The reward based properties are defined in combination with the *filter* operator to calculate maximum, minimum and average values of fault clearing time:

$$\text{filter} (\text{max}, R_{=?}^{\text{time}} [F \text{"healthy\_state"}], \text{"fault\_states"})$$

where *healthy\_state* is the state in which all the components are in the normal states and *fault\_states* is the state in which one or more components are faulted.

## 5 Case Study: A Power Distribution Substation

In order to illustrate the effectiveness and utilization of the proposed method for the formal reliability analysis of relay-based protective systems, we present the analysis of a typical power distribution substation, shown in Figure 4. The system consists of three transmission lines and a transformer with one relay associated with every component. The relays are assumed to be digital and equipped with the facilities of routine testing, self-checking and continuous monitoring. We have used the version 4.0.3 of the PRISM model checker running on a i7-2600 3.4 GHz processor, with 4 GB memory, as our verification platform.



**Fig. 4.** A Typical Power Distribution Substation

The transmission line model, shown in Figure 5, is obtained by including all three testing procedures in the basic model of Figure 2 according to the modeling approach, described in Section 3. The transformer model is the same as the transmission line model except for the difference in the transition rates and omission of states associated with temporary faults due to their rare occurrence in the case of transformers. In this system, it is assumed that the routine test and self-checking intervals will be the same for all of the four components. The transition rates, based on typical and experimental data, used in our experiments for these models are given in the Appendix. The states of both the transmission line and the transformer models are classified according to the labeling scheme, described in Section 4.2, as shown in Table 1.

**Table 1.** State Labels

State Label	Transmission Line	Transformer
Normal	1	1
Dependability	2,3,4,5	2,3
Unavailability	6,7,8,9,12,13,14,15,16,17	6,7,8,9,12,13,14,15,16,17
Security	10,11,18,19	10,11,18,19

The optimum value of routine test interval, without taking into consideration self-checking and continuous monitoring, is determined by minimizing the steady state probabilities of states labeled *security* and *unavailability* and maximizing

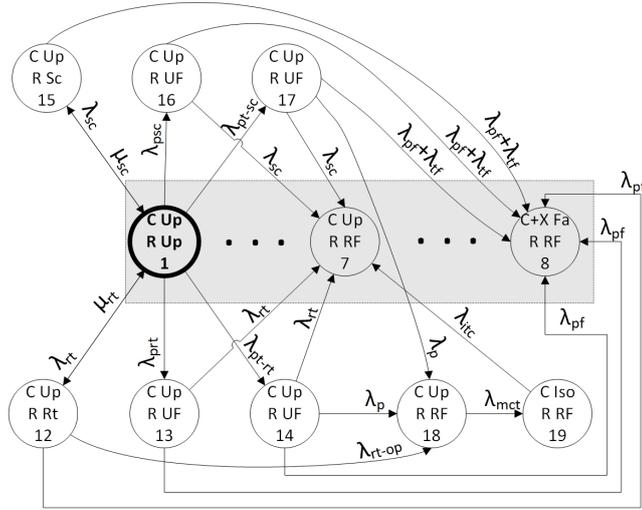


Fig. 5. Transmission Line Model

the steady state probabilities of states labeled *dependability* and *normal* in all of the four modules. This value comes out to be 1228 hours and the results for *normal* and *unavailability* states are shown in Figure 6.

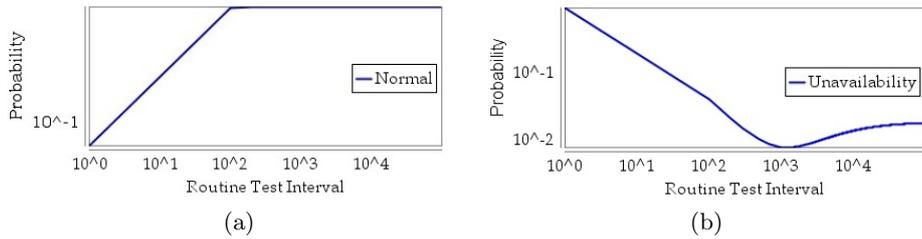
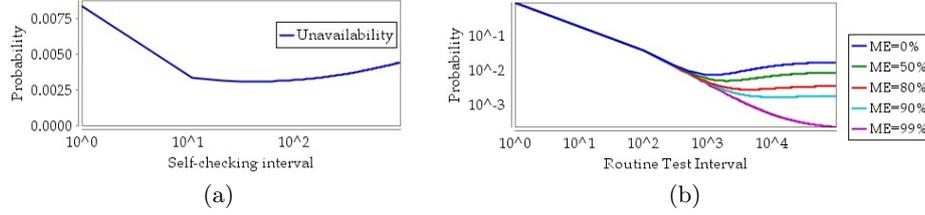


Fig. 6. Optimum Routine Test Interval (a)Normal (b)Unavailability

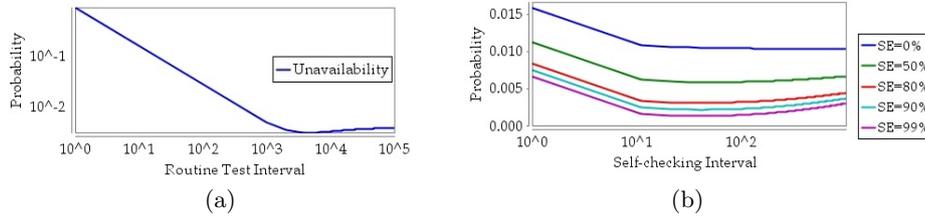
Self-checking (no continuous monitoring) is included in the system model by assuming the self-checking interval to be 20 hours and the self-checking effectiveness to be 80%. Optimum value of the routine test interval is then determined by minimizing the steady state probability of the undesired states. This value comes out to be 4398 hours which is greater than the case of no self-checking. The routine test is then assumed constant at its optimum value, i.e., 4398, and the optimum value of self-checking interval is determined, which is found to be 41 hours as shown in Figure 7(a). Continuous monitoring (no self-checking) can be included in the model of Figure 5 by adding a transition from state 1 to state 7 as mentioned in Section 3. The impact of including the continuous monitoring

only is that the optimum value of routine test interval comes out the same, i.e., 4398 hours but the probability of the system being in the *unavailability* state is now decreased as shown in Figure 7(b).



**Fig. 7.** (a) Optimum Self-checking interval (b) Impact of monitoring effectiveness

Figure 8(a) shows the probability of the *unavailability* state when both self-checking and continuous monitoring facilities are included simultaneously. Self-checking interval of 41 hours is assumed with its effectiveness of 50%. The effectiveness of continuous monitoring is assumed to be 30%. The optimum routine test interval comes out to be 4398 hours. This result is same as obtained previously with only self-checking or only continuous monitoring but the difference is that in this case, the probability of the *unavailability* state is lesser. Sensitivity analysis can also be performed for different values of self-checking effectiveness and the results are shown in Figure 8(b). The probability of being in the *unavailability* state decreases as the effectiveness increases.



**Fig. 8.** (a) All three testing procedures included (b) Sensitivity analysis

The permanent fault clearing time is calculated using the *filter* operator in combination with the reward properties. The minimum, average and maximum permanent fault clearing times have been found to be 1.005212032615363, 670.3588473481418 and 4026.7267711779245 hours, respectively. We can also find the steady state probabilities associated with the cases when the system is not in the *healthy.state* due to different number of components not in their *normal* states. Table 2 summarizes these results.

**Table 2.** Probability of the system not in *healthy\_state*

Module in normal state	Steady State Probability
Transmission line 1	$4.249870787 \times 10^{-9}$
Transmission line 1,2	$4.168868359 \times 10^{-6}$
Transmission line 1,2,3	$4.096023769 \times 10^{-3}$
Transmission line 1,2,3 and Transformer	$9.928568273 \times 10^{-1}$

The verification of properties, like the permanent fault clearing time and probability of the system not in the *healthy state*, can only be done using probabilistic model checking due to its exhaustive state-exploration. Thus, despite providing useful information, these properties, to the best of our knowledge, have not been verified by any other existing reliability analysis approach. Moreover, traditional techniques, like numerical methods and simulation, cannot match the precision of results obtained by the proposed approach for large systems like the one analyzed in this section. The complexity of the analysis can be judged by the fact that the verification of some of the properties, mentioned above, required exploring up to 116603 states and 983364 state-transitions. Finally, the plotting capabilities of PRISM were also found to be very handy.

## 6 Conclusion

This paper presents a formal reliability analysis approach for power distribution systems that use relays for protection. The main contributions of the paper include a modular approach to construct Markovian models for individual relay-based protected components of a power distribution system and using probabilistic model checking to analyze the reliability of power distribution systems at the system level. The paper presents a foundational model for a relay-based protected component that can have both component and relay faults. This foundational model can be extended to construct the models of most relay-based protected components and the paper presents its extensions for the most commonly used testing procedures, i.e., self-checking, routine test and continuous monitoring. Once the models of all the components of a power distribution system are obtained, they can be translated to the PRISM language to be executed in parallel to judge the reliability of the overall power distribution system using the PRISM model checker. The paper also identifies some key reliability assessment properties of power distribution systems that can be formally verified by PRISM. For illustration purposes, we presented the reliability analysis of a typical power distribution substation. The proposed approach has been found to be more accurate and scalable and it also allows us to verify many novel reliability aspects compared to the other existing reliability analysis approaches for power distribution systems. We plan to use the proposed approach to analyze the reliability of other power distribution systems while considering potential failures in the back-up relays as well. Moreover, the extensions of the proposed foundational model to include other components of power protection system, such as

current transformer (CT), voltage transformer (VT), Power Supply Unit (PSU), circuit breaker and the trip coil [1] are also worth exploring.

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## Appendix: Transition Rates

Rate Parameter	Transformer	Transmission Line
$\lambda_{pf}$	3 fault/yr	2 fault/yr
$\lambda_{tf}$	—	15 fault/yr
$\lambda_p$	0.01 failure/yr	0.01 failure/yr
$\lambda_{ext}$	0.0001 failure/yr	0.00001 failure/yr
$\lambda_{inr}$	0.0001 failure/yr	—
$\lambda_{exc}$	0.0001 failure/yr	—
$\lambda_{cp}$	0.000001 failure/hr	0.000001 failure/hr
$\lambda_{rct}$	—	10,800 operation/hr
$\lambda_{mct}$	30857.14 operation/hr	30857.14 operation/hr
$\lambda_{bct}$	21600 operation/hr	8640 operation/hr
$\lambda_{rt-op}$	0.001 failure/routine test interval	0.001 failure/routine test interval
$\lambda_{itc}$	0.5 operation/hr	0.5 operation/hr
$\lambda_{itx}$	0.5 operation/hr	0.5 operation/hr
$\eta$	0.1	0.1
$\mu_{sc}$	720 test/hr	720 test/hr
$\mu_{rt}$	1 test/hr	1 test/hr
$\mu_c$	0.1 repair/hr	1 repair/hr
$\mu_p$	1 repair/hr	1 repair/hr
$\lambda_{psc}$	$(1 - \eta)\lambda_p \times SE$	$(1 - \eta)\lambda_p \times SE$
$\lambda_{pmn}$	$\lambda_p \times ME$	$\lambda_p \times ME$
$\lambda_{prt}$	$(1 - \eta)\lambda_p \times (1 - SE - ME)$	$(1 - \eta)\lambda_p \times (1 - SE - ME)$
$\lambda_{pt-sc}$	$\eta\lambda_p \times SE$	$\eta\lambda_p \times SE$
$\lambda_{pt-rt}$	$\eta\lambda_p \times (1 - SE - ME)$	$\eta\lambda_p \times (1 - SE - ME)$